So what's new in Mathematica 5.0?

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If you are looking for a major visible change in what you see with Mathematica then I'm afraid you will have to wait until the next version. So just what is new? It's all under the hood where the core coding has been improved dramatically and the horizons of Mathematica have been dramatically improved. In the past, there have been many problems when developmental work was feasible with Mathematica but it was necessary to switch to other software for production runs – the aim of this upgrade is to eliminate the need to switch.

There have been numerous core changes to the latest version of Mathematica. This review can but give a flavour but hopefully it shows the dramatic improvement in performance.
Numerics

For many problems Mathematica 5.0 reveals substantial improvements in timing over version 4. For example[1] ,

	Version 5.0	Version 4.2
Solve 10000 random linear equations	0.471 seconds	4.396 seconds
Invert a 1000 by 1000 random matrix	1.432 seconds	20.55 seconds
Singular Value Decomposition 500 by 500	0.821 seconds	5.83 seconds

1 Computed on 500MHz laptop, 128Mbytes RAM

These improvements are due to

1. Modern implementation based on LAPACK and BLAS for machine precision numbers.
2. Optimized for modern microprocessors with big speed difference between processor and memory subsystem.
3. Taking advantage of vector instruction sets and multi-threading whenever possible.

The coding for sparse matrices has also been improved. For example, a matrix system of 10^5 equations would normally require $10^5 \bullet 10^5 \bullet 8 = 80$ Giga bytes, but the new sparse matrix coding, called densearrays, typically require about 4 Mbytes instead. In addition, one nice feature of 5.0 is the ease with which you can import and export several sparse array formats such as Matrix-Market and Harwell-Boeing.

Mathematica 5.0 also reveals major improvements in the capabilities of NDSolve; these include

1. Methods for n-dimensional initial value partial differential equations.
2. The ability to solve differential-algebraic equations (DAEs).
3. A larger collection of initial value solvers.
4. Support for the use of vector, matrix or general array variables.

To demonstrate all of these would not be productive. Instead we give just one example. The code needed to solve a Korteweg and de Vries equation in one spatial dimension subject to periodic boundary condition is shown with the solution in Fig 1.

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```
KdV[{α_}, t_, x_ ] := D[ψ[t,x], t] + α ψ[t,x] D[ψ[t,x], x]+D[ψ[t,x], {x,3}]==0;
Φ=S[{1,3,5},0, x] +S[{1,3/2,15},0, x]+
    S[{1,3,5+40},0, x] +S[{1,3/2,15+40},0, x]+
    S[{1,3,5-40},0, x] +S[{1,3/2,15-40},0, x];
(Φ /. x→0.) == (Φ /. x→40.);
s=NDSolve[ KdV[{1}, t, x] && ψ[0,x]== Φ && ψ[t,0]== ψ[t,40], ψ, {t,0, 10},
   {x, 0, 40}, Method→StiffnessSwitching];
```

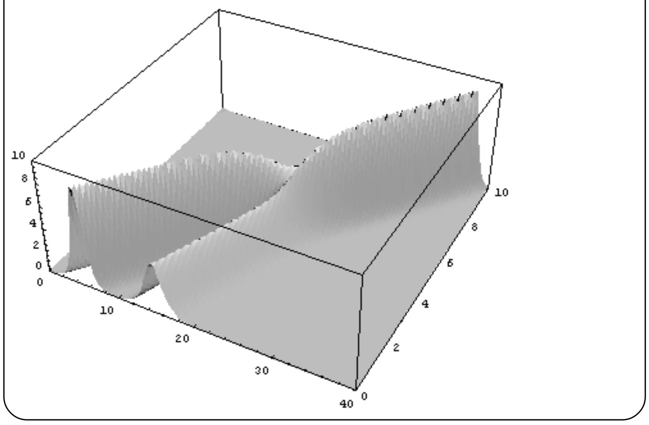

Fig 1 A solution of the KdV equation using Mathematica 5.0

Symbolics

As regards symbolic manipulation perhaps the most important improvements are

1. The possibility to reduce expressions inequalities and quantifiers, solve equations, and optimize functions and over different domains including complex, real and integers.

For example, solving $5^{2x-1} = 7^{3-x}$ over the real domain with

```
Reduce[ 5^(2 x-1) == 7^(3-x) && x ∈ Reals, x ]
```

produces

$$x == \frac{\text{Log}[1715]}{2\,\text{Log}[5] + \text{Log}[7]}$$

Solving over the complex numbers with

```
Reduce[ 5^(2 x-1) == 7^(3-x) && x ∈ Complexes, x ]
```

gives

$$C[1] \in \text{Integers} \;\&\&\; x == -\frac{2\,i\,\pi\,C[1] + \text{Log}[1715]}{-2\,\text{Log}[5] - \text{Log}[7]}$$

2. For differential equations, DSolve can now find all rational function solutions to systems of linear equations with rational coefficients and solve all linear systems of constant coefficient differential algebraic equations.

3. It is now possible to specify assumption explicitly when evaluating expressions, for example

$$\text{Assuming}[\sigma > 0, F = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}\,\sigma} \text{Exp}[-\frac{(x-\mu)^2}{2\sigma^2}]\, dx; \text{Limit}[F, y \to \infty]]$$

returns the value 1. One particularly nice feature is to be able to specify that a parameter is of a particular form, for example

```
Refine[Sin[k π],k∈Integers]
```

returns the value 0.

Import and Export

Mathematica 5.0 now supports an extended range of import and export graphic and file structures. These include

1. Sparse matrix formats such as Harwell-Boeing and MatrixMarket, as mentioned previously.
2. The export of XHTML has been improved to generate cascading style sheets to mimic look and feel of notebook style sheets.
3. Extended range of graphics and imaging formats, e.g. SVG, PNG and DICOM.

As an example, of the way Mathematica can now be used for substantial image processing the DICOM biomedical imaging format is now supported.

```
g = Import[ "MR-MONO2-16-knee", "DICOM"]
- Graphics -

Show[ g ];
```

Fig 2 Mathematica now supports DICOM biomedical imaging format

MathLink

There have been substantial performance improvement for *MathLink* over TCPIP and Windows. Since all *MathLink* bindings (JLink, netLink, PythonLink, etc) and almost all Import / Export formats rely on *MathLink* these will also improve as a result of improvements in *MathLink*. These improvements appear to be due to new SharedMemory protocols. In addition there is a new *.NETLink* binding of *MathLink* for Microsoft .NET and an improved *JLink* binding of *MathLink* for Sun Java.

Conclusions

Although there has been some redesign of the user-interface, for example new authoring palettes and a more extensive slide environment for existing style sheets, many users will perhaps not notice them at first. There has also been some change to the help browser interface to realign the current web and platform interfaces.

So why upgrade?

The answer is simple, if you were to take a top of the line sports car and fit a new engine that was ten times more powerful, that could go faster and longer on less fuel, then it is worth the cost. The same is true of this new version of Mathematica!

Book Review

Title: 1089 and all that: a journey into mathematics
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This delightful little volume is a mystery tour through some of the highlights of mathematics, both classical and contemporary. The small hardback volume, a format which has become popular for recreational mathematics and science books, opens with a chapter on Acheson's personal relationship with "the 1089 trick". The book continues with geometry and the obligatory graphical proof of Pythagoras' Theorem. Calculus, graph theory, the constants e and pi, through to modern applications, all squeeze into subsequent chapters. In addition to explanations of mathematical results themselves, the book also presents motivations for topics such as algebra and proof. The book closes with a chapter on the inverted linked pendulum, which Acheson has researched, and a finally a rationale for the imaginary numbers.

The style is chatty and crisp, and almost every page contains illustrations, which are well drawn and highly varied. Graphs, diagrams, photographs and cartoons support and enhance the text. In fact, the book itself is very short, and therefore does not contain much detail. However it doesn't set out to, and at least it does contain the appropriate equations where necessary. In fact, the author includes a lot of real mathematics if one cares to look closely, and the book is certainly not superficial. One is left with a slightly exasperating feeling that one wanted more: if that is true of all its readers then it is indeed a great success.

Why should a mathematician be interested in such a book? One reason is certainly that this is an excellent resource for those who wish to find inspiration for a school talk, open day presentation or master class. You probably also know someone who might like to borrow it.