Book Reviews

Title: Mathematics Teaching Practice, a guide for University and College Lecturers
Author: John H Mason
Publisher: Horwood Publishing, 2002, ISBN: 1-898563-79-9, £30

Reviewed by: Bob Burn
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This book is a gold mine. Its subtitle, *a guide for university and college lecturers* sounds modest enough until you begin to pick up what Mason sees the practice of mathematics teaching to be like. Mason himself is an active investigator of mathematical problems and he seeks to woo students into this world of active investigation and to do this as a means of studying conventional courses in pure and applied mathematics. With this intent he offers several hundred teaching strategies of which the majority will come as fresh and original suggestions to his intended readership. If a lecturer begins to try out some of Mason's suggestions (and Mason makes the realistic remark that no teacher can work on the improvement of more than two aspects of his/her teaching at once) he will find himself hamstrung unless he himself stays active in the pursuit of undergraduate mathematics. For every one of his students, the subject is new, and that novelty, the teacher must share in.

This book contrasts starkly with *How to teach mathematics* by Stephen Krantz (AMS, 2000) where the knowledge-competence of the lecturer is taken for granted. Mason's book describes undergraduate mathematics as an unbounded field both because the questions and answers of each year's students generate a crop of new ideas, and because the history of the subject (that one delves into gradually with age) has its own dimensions to enhance awareness of mathematics and mathematicians.

In the 1990s a series of teaching texts *53 Interesting Ways to teach...* were published. The mathematics text was written by Ruth Hubbard. In that series most of the 53 ways were likely to be used by a good lecturer who was sensitive to student learning. Mason's book proposes at least 253 interesting ways to teach mathematics and of these I doubt whether any lecturer, other than Mason himself, has ever considered using half of them. The originality of Mason's proposals is breath-taking. Although he writes simply and clearly, the teaching suggestions come thick and fast. Don't expect to be able to reflect on more than one or two pages at a sitting.

Mason's illustrations cover algebra, analysis, probability, mechanics and other aspects of applied mathematics, and while this variety underlines the universality of his proposals, it may give the false impression that a lecturer who is Mason-competent as a teacher of algebra (say) will necessarily be Mason-competent as a teacher of analysis. The pursuit of Mason-competence in any one area of undergraduate mathematics teaching should not be thought to need less than a ten year programme. I would urge those who have been challenged and inspired by this book to construct specialist versions of it: *Algebra Teaching Practice*, or *Analysis Teaching Practice* or *Combinatorics Teaching Practice* or *Probability Teaching Practice* or others. The application of Mason's ideas to each area is non-trivial and publishable. In this more specialist context it should be possible to address the place of mathematical axioms in learning - a subject which Mason avoids.

As a final comment, it is a pleasure to note that the book is liberally spattered with stimulating quotations. Here are two.

> *The authority of those who teach is a hindrance to those who would learn.* (St Augustine)

> *The hardest part of teaching by challenging is to keep your mouth shut, to hold back.*
>
> *Don't say, ask!*
> *Don't replace the wrong A with the right B, but ask "where did A come from?"*
> *Keep asking "Is that right? Are you sure?"*
> *Don't say "no", ask "why?".* (Paul Halmos)

Title: Mathematics: A Second Start, 2nd edition
Authors: S Page, J Berry and H Hampson
Publisher: Horwood Publishing, Chichester ISBN 1-898563-04-7

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This book is a second edition, updated by Howard Hampson and John Berry, of Sheila Page's first edition published sixteen years ago. The new authors state that they have attempted to retain the original aim of the book; that is "to provide a second start at mathematics for those students who 'never could do maths'". That the book originally dates from another era is shown by the first author's comments in the introduction to the first edition where she states that the book is for those who have missed their way at school 'either by absence from some of the O-level course, or by having too many changes of masters or schools'.

The mathematical content of the book begins with the set of real numbers, covering such things as commutivity and associativity, HCFs and LCMs, rules of precedence and use of brackets. It then progresses through number skills covering directed numbers and fractions into a substantial section (11 chapters) on algebra followed by a thorough coverage of calculus during which time trigonometric, logarithmic and exponential functions are covered. The final three chapters cover statistics and probability.

The book is not a rigorous treatise on mathematics. The authors do not intend it to be. They declare this quite openly. 'The treatment used is by no means exhaustive and we try to appeal to intuition wherever possible. We prove many things by using examples rather than using general proofs.' The sentiment of such a statement is clearly appropriate for a book such as this, but it is unfortunate that the authors expressed it in this way. Many students are already confused about what is meant by proof. It is not unusual to find students who think that they have proved something is true because they have found a number of examples where the result holds. A statement such as 'we prove things by using examples' reinforces such a belief.

The authors also seem to assume that the readers will be at ease with the use of counter-examples. So, the all too frequent error that

$$1/(a+b) = 1/a + 1/b$$

is dismissed (in the section on arithmetic of fractions) by a single example that

$$3/(6+4) \neq 3/6 + 3/4.$$

I doubt if many of the readers will realise the significance of this, even though the authors do state that this is very important in algebra.

The pace of the book seems to be about right and there are plenty of worked examples for the students to use as models when they attempt the many exercises that are included in the text (sprinkled throughout each chapter, not just at the end). The answers to the exercises are included so that students can check that they have solved the problems correctly.

As the Director of a University Mathematics Support Centre I regularly encounter students who say that they 'never could do maths' and so the aim of this book appealed to me. We often encourage students to forget bad experiences that they may have had with mathematics in the past and to make a fresh (or second) start. On the face of it this book should make an ideal resource for the Support Centre. Unfortunately the book did not quite live up to the hopes I had for it.

The book is littered with errors and although the vast majority of these are typographical they tend to occur in rather unfortunate places. Many of them will cause confusion in the mind of the reader, who almost by definition of the target audience, will be lacking in confidence and more likely to think that they have misunderstood what they have read rather than what they have read is wrong. So, for example, a worked example to illustrate how to deal with negative signs outside brackets says

$$-(x+y) = -x + y.$$

From the context it is obvious to the experienced eye that what was meant was $-(x-y)$, but to the struggling reader this may not be so clear and the very failing the example was meant to address has actually appeared as correct. If this were an isolated incident it would not be a major problem – indeed it is difficult for a book of this size to be error free. It is the regularity of such errors that is the problem.

Throughout the section on the normal distribution the text repeatedly makes statements such as

$$p(t=1) = 0.3413$$

Clearly (to someone who already understands what probability distributions are) what is meant is

$$p(0<t<1) = 0.3413$$

However, the sheer repetitiveness of this error will leave students new to the concept at best confused and at worst with a completely incorrect understanding of what a probability distribution is and the information contained in tables of the normal distribution.

The authors fall into the trap of referring to mathematical ideas which the book has not yet covered. So the first chapter refers to the numerator and denominator of a quotient (these are defined in the third chapter) and square roots (covered in the second chapter). And the last chapter refers to 'interpolation' which is not mentioned anywhere else. When the book was originally written 16 years ago, for those whose knowledge extends 'to O-level mathematics', it may have been reasonable to assume that the readers would be familiar with such ideas, but it is not safe to assume such knowledge in today's students needing to make a second start in mathematics.

Overall I have to admit to disappointment with the book – I will not (as I had hoped) be adding it to the resources of our Mathematics Support Centre.

Title: Multimedia Tools for Communicating Mathematics (MTCM2000)
Editors: Jonathan Borwein, Maria Morales, Konrad Polthier and Jose Rodrigues

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This book arose from presentations at an international workshop organised at the Centro del Matematic e Aplicacoes Fundamentais as the University of Lisbon, in November 2000, with the collaboration of the Sonderforchungsbereich 288 at the University of Technology in Berlin, and of the Centre for Experimental and Constructive Mathematics at Simon Fraser University in Burnaby, Canada. The meeting took place under the auspices of the Socieda Portuguesa de Matematica and the European Mathematical Society, and was also sponsored by a special grant from the Fundacao para a Ciencia e a Tecnologia of Portugal. The workshop gathered fifty-seven participants; there were twenty-nine presentations and a round table discussion.

The text is the proceedings of the above meeting, which considered the scientific methods and algorithms supporting an array of multimedia tools, of their limitations and the underlying mathematical problems. Besides new tools; new mathematical algorithms and data structures are needed for online mathematics.

In addition the conference proceedings give a significant number of online sources that are already available or that will appear shortly. For example, Neil Sloane's server of integer sequences, Finch's Constants at MathSoft, and the newly established EG-Models server in Berlin. Full details are given in the text. The proceedings contain the text of all twenty nine presentations. In addition, it includes a CD-ROM with dynamic examples of many of the ideas fundamental to the meeting. Of particular interest are the following papers:-

A Virtual Reconstruction of a Virtual Exhibit (*Thomas F Banchoff and David P Cervone*), pages 29-38. This gives details of a travelling exhibit "Para Alem da Terceira Dimensao" which was the product of international collaboration between the US and Portugal. Inspired by a virtual art gallery "Surfaces Beyond the Third Dimension", is a classic example of how visualisation plays a key role in the understanding of mathematics.

Visual Calculus *(Lawrence S Husch),* pages131-140. This presentation gives details of the Visual Calculus Project based at Knoxville Tennessee that includes a large number of software packages to help visualisation and the exploration of key concepts in freshman calculus. In particular the presentation discusses many of the techniques that were used to develop this material, some of the successes and also some of the problems which arose.

Dynamic Geometry (*Gilles Kuntz*), pages 221-230. This presentation explores the development of dynamic geometry in teaching and how it can be used to stimulate the development of WWW sites. In particular the project "Cabri-Java" is discussed, in which a number of Java applets are considered.

Minimalistic Tools for Mathematical Multimedia (*Erich Neuwirth*), pages 231-240. This overview presented a wide array of software tools and how they can be used to create multimedia units that allow the visualisation of mathematical concepts.

These proceedings cover an enormous spectrum of both techniques and approaches, a diverse array of pedagogical approaches and some extremely useful discussion. The presentations, all approximately ten pages in length, are ideal to "dib" into and are insightful. As a resource, these proceedings should be on the desk of anyone with either a desire or need to develop multimedia tools for teaching and visualizing mathematics at any level.