Ride the Rollercoaster with the Mathwise Calculus Cluster

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If NAG, the company which markets Mathwise, is looking for a logo for the calculus cluster, they could do worse than choose the corkscrew rollercoaster at Alton Towers. The module 'Applications of Differentiation' contains a video clip of the ride and then uses a wireframe animation (shown in Figure 1) as a way of introducing the ideas of local maxima and minima. This is a creative and stimulating example which should help to gain students' interest.

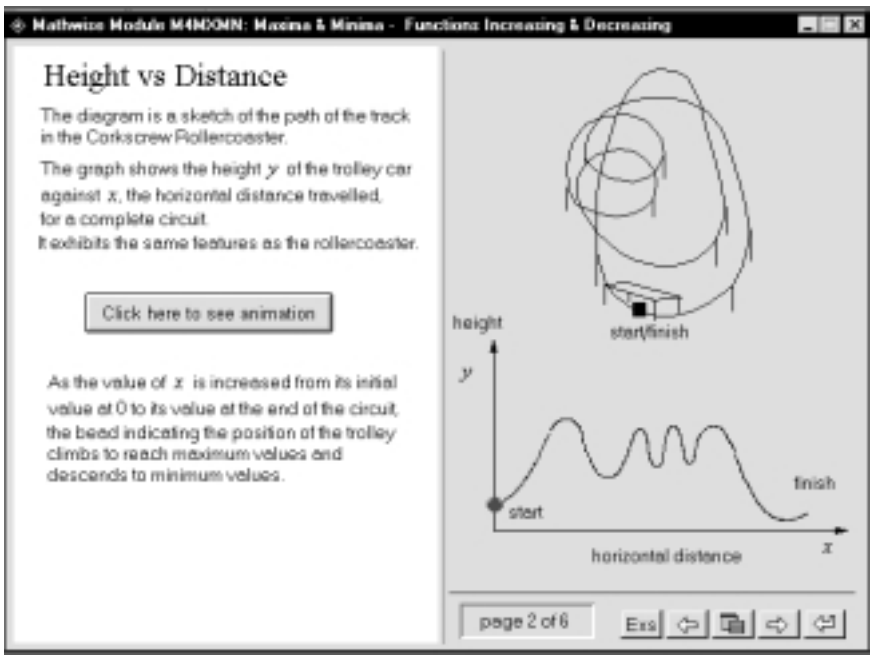

Figure 1: Wire Frame Animation of Alton Towers Corkscrew Rollercoaster

There is another sense in which this would be an appropriate logo for this package, although this would probably make it not quite so appealing to NAG. A rollercoaster is characterised by its highs and lows and unfortunately the same is true of this package. In some places it is an excellent, imaginative and appealing resource for learning about calculus, whilst in others it is little more than an on-screen book, heavily reliant on static algebraic explanations.

Some of this variation is due to the nature of the material. It is much easier to have visually exciting, interactive activities describing the concepts of derivatives than for outlining the quotient rule for differentiation. Having said that, there is also a sense that some (perhaps most) of the variation is due to the fact that the modules are written by different authors and their own personal style comes through (despite the broadly standard layout).

Content

In *Maths&Stats* Volume 9 Number 3 David Begg reviewed the pre-Calculus cluster of Mathwise v2.1 [1]. That review contained much general information about the structure of Mathwise, how it can be customised, the general purpose resources, navigation and assessment. This article will, as far as possible, avoid duplicating the comments made there on these general features of Mathwise and concentrate on the content of the modules which make up the Calculus cluster.

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The Calculus cluster consists of five modules covering

- Concepts of Differentiation
- Rules of Differentiation
- Applications of Differentiation
- Introduction to Integration
- First Order Ordinary Differential Equations

The 'Using Mathwise' section of the help file informs us that each module contains the equivalent of about 5 hours of lectures. The integration module is labelled as a double module and it certainly is at least that. The module gives a very thorough coverage of integration from Riemann sums, through the fundamental theorem of calculus to the main techniques of integration (substitution, parts and using partial fraction for rational functions).

Structure & Navigation

The basic structure within Mathwise is the Module. By clicking on the courseware button on the entry screen followed by the modules button on the resulting sub-menu the user is presented with a list of modules. Each module is divided into a number of learning units and these can be accessed separately from a module contents page like the one shown in Figure 2.

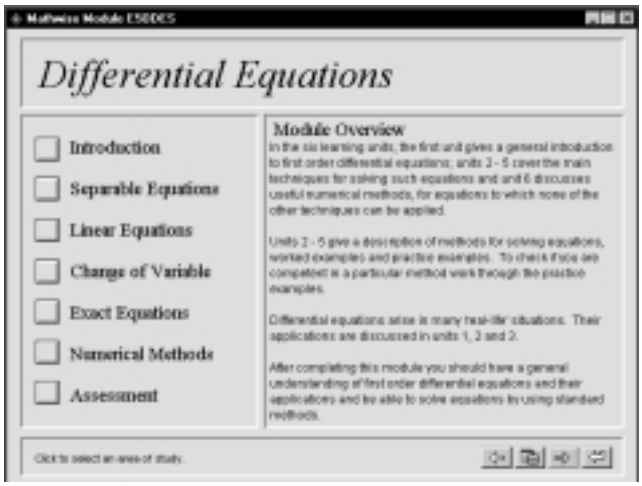

Figure 2: Contents Page of the First Order Ordinary Differential Equations Module

A learning unit is then further broken down into sections. In the differential equations module when a learning unit is selected the list of sections appears in the right half of the screen and a section can be selected from this list. In other modules the list of sections appears on the first page of the learning unit which is sometimes entered directly once it has been selected on the contents page or which sometimes requires a further button click to 'Open Selected Unit'.

Navigation is primarily next page, previous page or return to the next level up, using the buttons at the bottom right of the screen. I found it irritating in the differential equations module that when I completed a section there was no next page button to take me to the first page of the next section. Instead I had to return to the contents page shown in Figure 2 and select the next section from there. This seemed unnecessary given that many of the sections are very short (one or two pages).

The Modules

The module on 'Concepts of Differentiation' was my personal favourite. It contains many interactive animations which enable the learner to explore various concepts relating to differentiation, such as local linearity and chords approaching tangents. There is plenty of self assessment material available for the reader to check how much material is being understood. I particularly liked the exercise in the 'Sketch derivatives' test which presented the graph of a function and then required the reader to sketch its gradient by dragging the handles on a jointed line to produce the gradient curve. An example of this is shown in Figure 3.

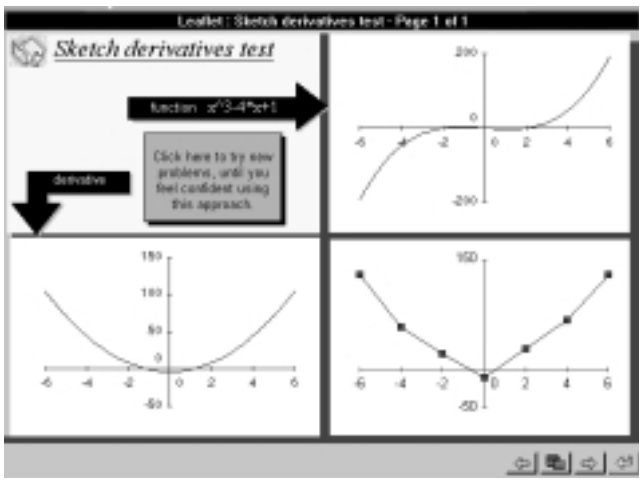

Figure 3: Gradient sketching exercise

This is an excellent example of using the technology to do what it is good at and what it would be hard to do any other way and whilst personally I would have preferred the correct gradient to have been superimposed over my attempt (so that I could more easily see how close or otherwise I had been) this hardly detracts from a first-rate exercise.

Another nice feature of this module was the tangent animation shown in Figure 4. The reader can select a function of their choice, plot its graph and then, by clicking the animation button, see the tangent move along the displayed curve. This effectively visually

reinforces the information in the text on the left hand side of the screen. This animation worked for a wide range of functions but for no apparent reason drew completely the wrong tangents for the function exp(-x^2).

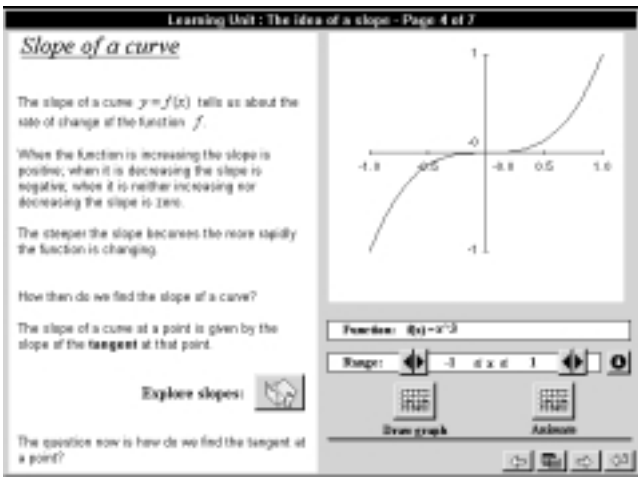

Figure 4: Tangent Animation

As the modules have been written by different authors it is not surprising to find that there are inconsistencies of style. Most of the time these are of no consequence but there are times when the inconsistencies move beyond issues of style to ones of approach. This is perhaps most clearly seen if the two modules Rules of Differentiation and Introduction to Integration are compared.

The Rules of Differentiation module appears to take what might be called the minimalist approach. All the rules are covered but with very little fuss and most of the time the user has little to do other than simply read the material on the screen. There are a few buttons to click to bring extra material on screen and some exercises to do, but there is little by way of true interaction. However, there is one high spot: proving the product rule. Here advantage is taken of the graphic power of the computer to illustrate the proof and to allow it to be stepped through in a systematic fashion. But there was little beyond this that grabbed my attention and I found it the most disappointing module of the cluster.

The Introduction to Integration module, on the other hand, takes what might be called the maximalist approach. This is not just in terms of the use that is made of graphics and interaction but also in terms of what material is included. As I understand it, Mathwise was originally conceived as being support material for engineering students. I do wonder how many engineering students are taught about Riemann sums in the depth they are covered in this module. I suspect that only the most highly motivated students will work through some of the learning units of this module. That in itself is not a problem, I just hope that having this material in the first section does not deter others from going through to the more technique oriented later parts of the module. If I were to use this module I would feel that I would need to write a study guide indicating which parts students could skip over and which parts they should focus on.

The other modules lie somewhere between these two extremes. The Ordinary Differential Equations module strikes a commendable balance between providing enough material to cover the subject whilst not overwhelming the user with a sea of information. Many of the screens were attractive and most of the screens which ended up full were built up in stages. The screen shown in Figure 5 is one such screen. The user has, by a series of interactions, built up the right hand side of the screen by working through the steps of the method as outlined on the left hand side of the screen. I would be happy to let my students use this module with very little additional guidance other than to warn them not to be too disappointed when their attempts at the practice examples are marked wrong. Every example I tried was marked with a cross, even when the answer I had given was exactly the same as what was subsequently revealed as the correct answer.

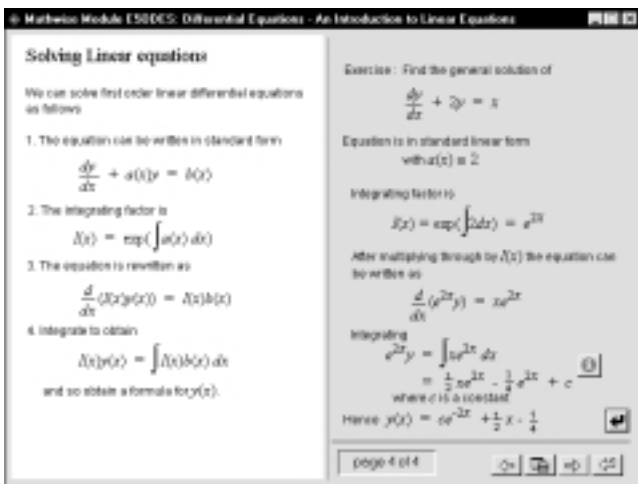

Figure 5: Solution of Linear 1st Order Differential Equation

Some Comments

There is much in the Mathwise Calculus cluster that is well thought out and well implemented. Many of the interactive activities are stimulating and, if used in a thoughtful rather than a button clicking manner by students, should enhance learning. However, the package cannot be used blindly. Careful thought needs to be given to which parts of the package are the most

useful to particular student groups. If study guides which directed students to appropriate sections were to be produced the package could be useful to both more able and less able students, both specialist mathematician and non-specialist.

The reason I feel that the study guides are necessary is that in many places it looks as though the modules are written by able mathematicians for able mathematicians. Many of the modules rely heavily on substantial tracts of algebra. Frequently topics are introduced in generality and only after the key result has been proved are some examples given. My experience with engineering undergraduates has been that they think the other way round. They like to start with specific examples and generalise to the key result. As we are all aware, students entering higher education do not have the same level of algebraic skills as their counterparts 10 years ago and so relying too heavily on these skills is a risky strategy.

When I was first asked to review this package I was supplied with a pre-release version which was riddled with bugs. Later on I was supplied with the commercially available package and I am pleased to report that many of these have been fixed. Hopefully those remaining, which are primarily presentational, will be eliminated in future updates.

Conclusions

As indicated at the start of this article, in my opinion the Mathwise Calculus cluster is like a rollercoaster with plenty of highs and lows. Its highs are exemplary and its lows are not disastrous. In many places the authors' have exploited the potential of the computer and used visual presentation and interaction to retain the user's interest and to present material in clear and instructive ways. However, in my opinion, there are occasions when the material presented is too static and rather more heavily dependent on algebra than may be wise.

Reference

{1} Begg, D, Review of Mathwise v2.1 (Pre-Calculus Cluster), *Maths&Stats*, 9(3): 15-18

Response to review of the Mathwise Calculus Cluster
Mike Hooper, NAG

We are grateful to the reviewer for revisiting the Mathwise Calculus Cluster, after his earlier experience with the less than perfect alpha version, and are pleased that he notes the improvements in the package. As he comments, much of the variation in presentation is due to differences in authors' styles; he may not be aware that this is also due to their use of disparate authoring technologies. Fortunately, due to the efforts of the systems staff on the Mathwise project, much of this variation has been hidden from view.

At the time of writing, we have generated a correction to one of the problems on which the review comments (the drawing of tangents to negative exponential curves) and are attempting to resolve the other reported problem.

We should like to emphasize that the material is not, as the reviewer hints, intended only, or even primarily, for engineering students; it is designed to be suitable for anyone wishing to learn more about calculus. This includes those employed in industry but continuing their education, as well as students in other undergraduate disciplines. The many exercises and "assessments" which are built in to the package can provide self-assessment for an individual user or form the basis for assessment by a tutor in a class environment.

It would, perhaps, be appropriate to mention here that NAG's involvement with Mathwise has been motivated by a desire to make potentially very useful material, generated by the project, available to as wide an audience as possible, rather than by any commercial considerations. Any revenue derived from Mathwise products is intended principally to help the project to develop further and to cover immediate distribution costs. NAG has been happy to contribute the services required for quality assurance and assembly of the material, in both the Pre-Calculus and Calculus Clusters, as we are committed to the overall aims and objectives of the project.