AUTOGRAPH Version 2.0 for PC

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AUTOGRAPH is a powerful interactive graphical tool for mathematical education in two main areas;

1. Cartesian transformations, equations and inequalities, including calculus and differential equations.
2. Statistics, including discrete and continuous distributions and confidence testing.

As I see it, it has two main roles in higher education teaching;

- In a lecture or tutorial, as a tool for demonstrating important concepts as they are introduced and developed.
- In a laboratory based sessions, where students are given pre-set examples which they can investigate to improve their understanding.

It would also be possible to allow students to use the package freely to investigate problems; but this would involve an overhead in training them to use AUTOGRAPH correctly.

The main way that AUTOGRAPH is superior to other packages with graphical capabilities is in its use of natural Windows techniques such as selection, dragging and pop up menus to control the exploration of mathematical concepts. This design enables rapid mastery of the package, as similar techniques work well in many different parts of the application. This will be appreciated most fully by trying the package for yourself; but here are a few examples.

Differentiation from First Principles

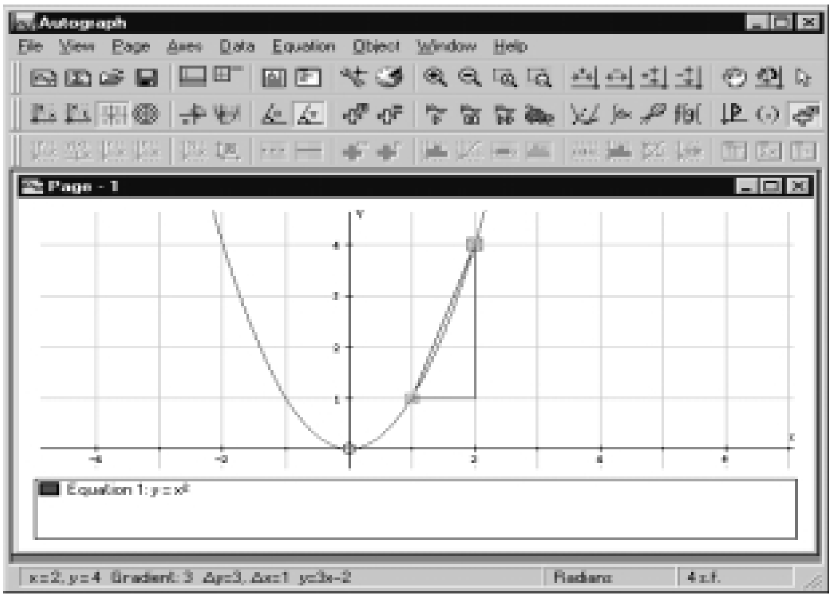

Fig 1: A simple to create graph in AUTOGRAPH

The above graph is created very simply in AUTOGRAPH; the sequence required after starting up the package is shown overleaf:

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1. Type ‘↵y = xx↵’. (‘↵’ is the enter key).
2. Click the cursor mode button (third from the right on the second toolbar).
3. Click twice on the graph to insert two points.
4. Use shift-click to select both points.
5. Right click, and select ‘gradient’ from the pop-up menu.

This takes longer to read than to do; but it is also possible to store the resulting graph in a file for later use. The diagram can now be manipulated in many ways, just like a picture on a board can not. Here are a few of them;

- The full secant line can be added just as simply as the gradient triangle, if desired.
- One of the points can be dragged along the curve; as it is moved the values of Dx, Dy and the gradient are automatically updated.
- By clicking on a button on the top toolbar, it is possible to zoom into the graph.
- By using the animation controller, the lower point can be caused to approach the upper by clicking a button, or at a range of speeds.

Integration

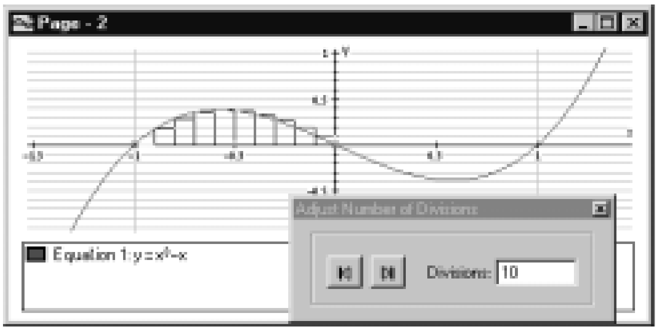

Fig 2: Exploring the limiting value

For this example, we entered the graph in a similar manner to the previous example, and then highlighted two points (“cursors”) on the graph. Right-clicking brought up a pop-up menu including a “find area” option, which displays a dialog box to choose one of four methods of numerical integration. After this has been chosen, and the parameters set, the selected elements of area are drawn on screen, and the total area displayed at the bottom of the AUTOGRAPH screen. The limiting value can be explored by increasing the number of divisions; and the area under investigation can easily be altered by dragging either cursor on the curve.

Chaos and Iteration

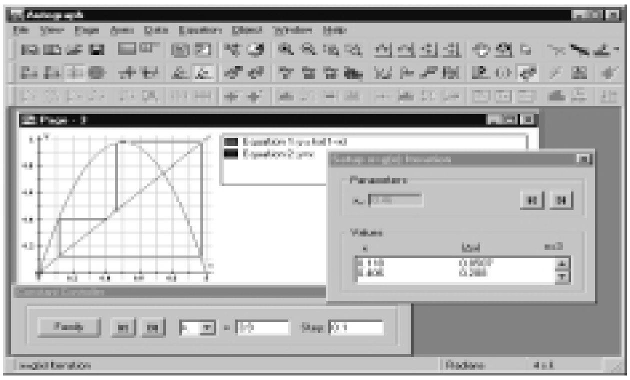

Fig 3: Threefold iteration on logistic map *y=kx(1-x)*

In this picture, we have set up a threefold iteration on the logistic map *y = kx(1-x)*. The initial value x_0 for the iteration can be controlled by dragging the cursor on the line *y = x*; the value of *k* can be altered using the constant controller window. By dragging the cursor to *x* = 0.45 we can demonstrate the existence of period three point. More impressively, if we reduce *k* to 3.8 we can give an informal demonstration that no period three point exists by dragging the cursor up and down the line *y = x*, and watching the L-shape orbit just fail to materialise. The famous butterfly effect can be investigated by setting up two iteration objects with nearby starting values. The starting cursors can then be grouped together so that they always keep the same distance apart. As the pair of dragged up and down the line *y = x*, a simultaneous visual and numerical observation of the butterfly effect is possible.

Descriptive Statistics

It is easy to import previously defined data into AUTOGRAPH from a spreadsheet by cutting and pasting; it is also a simple matter to generate random data by means of a user selected distribution, including binomial, rectangular, Poisson and normal varieties. Data can be presented as histograms, cumulative frequency curves, box-and-whisker diagrams and dot plots. In contrast to some general purpose packages, care has been taken to get the mathematical details right, as in the proper distinction between histograms of discrete and continuous data.

Conclusion

There is not enough space to illustrate the huge range of features in Autograph; I would refer anyone who is interested to the Autograph web site, http://www.autograph-maths.com. The site also gives details of pricing and ordering information. At the time of writing this review, prices ranged from £50 for a single-user license to £400 for an institutional site license. In such a complex piece of software there are bound to be a few glitches, and I have reported a number of these directly to the authors. The intention is that as more people go off the beaten track in their exploration of this software, and discover unintended results, the most recent version of the executable file will be made available to those who have bought the package via the web site.

However there is a wealth of material presented on the Autograph CD which can be used with very little difficulty. The examples cover a wide range of topics treated on undergraduate courses where mathematics is required, and would enrich the experience of many of our students immensely.

Book Review

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Introduction to Real Analysis, Robert G Bartle and Donald R Sherbert, John Wiley 1999

This is the third edition of this popular text on Real Analysis by two well-established authors. In this new edition there has been a small amount of revision in the exposition, some new exercises have been set and there is a totally new chapter on the generalised Riemann integral.

The content is fairly typical of an analysis text. Topics covered include sequences, series, limits, the derivative, sequences of functions and infinite series. In addition there are chapters on the Riemann integral and the generalised Riemann integral. The authors' justification for this latter chapter being that the generalised Riemann integral (also known as the Henstock-Kurzweil integral) has a greater range of application than the Lebesgue integral. There are appendices on methods of proof, countability, approximate integration and integrability criteria. Each section of every chapter has a generous set of exercises of varying difficulty, although the harder exercises are not starred and there are no slightly longer "project exercises" which characterise Bartle's solo text "The Elements of Real Analysis". About half of all exercises have answers or at least hints to the solution. (A teacher's manual, with almost all solutions, is available upon request to the publisher).

The quality and detail of the exposition is excellent throughout the text with care given to explain the objectives and purpose of each section. For example, when discussing the convergence of a sequence, considerable effort is employed to direct the reader's attention to the "ultimate behaviour" of the sequence via the idea of its tail. In the section on countability the authors give two examples of diagonal mappings to illustrate the countablity of infinite sets. They provide the usual construction of a surjection from **N** into the rationals **Q** and also construct a bijection between **N** and the Cartesian product **N x N** .

Chapter 10 gives an exposition of the generalised Riemann integral which was developed in the late 1950s and includes both the Riemann integral and the Lebesgue integral as special cases. Working through this chapter is not as tough as it could be because the development mirrors that of the earlier chapter on the Riemann integral, but uses the idea of a gauge to control the variable fineness of the partition of the interval of integration.

This text takes the reader far beyond the realms of the common American tomes on calculus and analysis yet shares a common origin. It will prove valuable to those who need to work in the most general or abstract setting and require a readable, useful reference at a level that encompasses both undergraduate and postgraduate contexts.