# Something that worked for me...

*Title:* **Eigenvectors via Excel**

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Our students meet eigenvalues and eigenvectors in year 1 of their course. A couple of years ago, I wrote an EXCEL spreadsheet which enables them to find the eigenvalues and eigenvectors of a 2x2 matrix experimentally.

The idea, at its simplest, is that the spreadsheet allows the student to supply a 2x2 matrix ($A$, say) and then to rotate a unit vector, **u**, about the origin, with the spreadsheet showing, at every stage, the vector **u** and the transformed vector $A$**u**. The student then merely rotates **u** until the original unit vector and the transformed vector $A$**u** point in the same direction. The components of vector **u** can then be read off from the spreadsheet, as can the magnitude of $A$**u** (which, since **u** is a unit vector, gives the magnitude of the eigenvalue).

I have used the spreadsheet with some of our students and it has been well-received. Some of the benefits of using this approach are listed below.

i) The student is in control of his/her own learning.

ii) He/she reads a description of what is meant by eigenvalues and eigenvectors and, within minutes, has a graphical illustration of the ideas and is finding eigenvalues and eigenvectors experimentally.

iii) The idea of an eigenvector as being one for which multiplication by the matrix is equivalent to multiplication by a scalar is presented forcefully.

iv) A matrix such as $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, for example, is seen to represent a rotation, with the transformed vector remaining doggedly 90° behind the original vector. The idea that a matrix may have no real eigenvalues is thus illustrated and so it comes as no surprise when the student (later) encounters a characteristic equation which has no real roots.

v) We make use of the limited precision to which the spreadsheet works. For example, if the spreadsheet seems to indicate that $\mathbf{u}=\begin{pmatrix} 0.71 \\ 0.71 \end{pmatrix}$ is an eigenvector with corresponding eigenvalue given by $|A\mathbf{u}| = 2.99$, then the student is led naturally to conjecture that the eigenvalue might be 3 and that the vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ - or even $\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$ - is an eigenvector. He/she can easily test this conjecture. This example appears in the hand-out given to the student – but the situation arises frequently.

vi) It's a bit different – and fun.

## Editors Note...

We would like to thank Michael for supplying the worksheet and excel files mentioned above. To view or download the worksheet handed out to the students and a copy of the excel file please go to our web pages at:

Worksheet (PDF/Word97)

**http://ltsn.mathstore.ac.uk/newsletter/may2002/mgrant/eigenwork.pdf**
**http://ltsn.mathstore.ac.uk/newsletter/may2002/mgrant/eigenwork.doc**

Excel file

**http://ltsn.mathstore.ac.uk/newsletter/may2002/mgrant/eigen.xls**