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# Review of *Introduction to Groups*

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**I**ntroduction to Groups is the first of a series of learning resources designed for individual study of some topics in university level mathematics. Written by David Saunders and published by Symplekta, it aims to provide an introduction to the subject of Groups for Mathematics and is suitable both for University students and for those studying Further Mathematics at A level. The author has also written a follow-up (doing some serious group theory for 1st/2nd year university students) and is now starting on a disk introducing matrices.

The material consists of a CD-ROM and requires a PC with a sound card and a screen with 16-bit or 24-bit colour, as colour is used throughout to enhance the explanations and, with only 256 colours, some of the text may be illegible. Windows from 95 onwards are recommended.

There is no accompanying booklet, just a card insert in the CD jewel case with some basic instructions. As we shall see, the material does not really need any hard-copy support, but a list of the pre-requisites (a knowledge of sets and binary operations) could usefully have been given here.

The CD consists of an hour or more of audio tutorial (depending on the user's speed of progress), animations and interactivity, exercises and a hyper linked glossary. It uses numbers, permutations and the symmetries of geometrical objects to motivate the introduction of group axioms. Concrete examples of small groups are used to illustrate the ideas of subgroup and isomorphism, and a statement of Lagrange's Theorem is used to motivate the definition of a coset. The proof of Lagrange's Theorem is covered, together with that of Fermat's Little Theorem as a corollary.

I gave this disk to two students: One is studying Further Mathematics at A level in order to read Maths at University and all the contents of this CD are on the specification for her P6 module; the other student is in his second year reading Maths at university. Their comments are incorporated in this review.

## ***Getting Started***

On loading the CD, the front cover appears and, as a chord sounds; we are pointed to a button to 'Enter'. The next screen consists of a page headed 'A number game' on which some numbers can be dimly seen; there is a menu down the left hand side and arrows at the bottom. However the tutorial immediately poses a problem. Expecting some lead-in, one looks in vain for a helpful 'How to Start' or 'Next' and after a little trial finds that it is necessary to go to the 'Help'. This is clearly set out but to discover everything required entails going through several pages in a rather meandering route and it would have been more helpful to have given all the essentials in a clear introductory page before the start of the actual tutorial. Alternatively, navigation through the Help section could be made simpler: On clicking 'Using the tutorial' an explanation plus a list of three further topics is given. Each of these leads to further menus consisting of combinations of any two or three of the six headings available. A full list of the topics initially would save much searching back and forth to ensure that one has not missed one of them out! Both of my student guinea pigs found this lack of initial guidance irritating when they were eager to begin.

Once found, the instructions are clear and moving around within the tutorial is very easy and flexible, using either the mouse or the keyboard. Movement back and forth between pages of a section or between sections or pausing is

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direct, while a 'rheostat' enables moves backwards or forwards of just a few seconds. The typeface is clear and each page is talked through orally, imposing a fixed speed for these sections but generally the student can go through the tutorial very much at his own pace. The voice which explains each section is a good one, clear, unaccented and at a pitch which is easy to listen to. The first page starts quite slowly and I was afraid that it would become frustrating but was relieved to find that, after explaining two examples to establish the ideas, it swiftly moves on. I found the voice comforting, a clear teacher explaining each new point but then setting small challenges to the pupil.

### The Tutorial

The contents menu, on the left of the first screen, lists: A number game, Some examples, The group axioms, More examples, Subgroups, Isomorphic groups, The order of an element, Lagrange's Theorem, Cosets and The proof of Lagrange. These can be accessed in any order but, initially, it makes sense to follow them through in the order given. The exercises too can be done at any stage but I soon found that, in order to even start each one, the student would have to cover the whole of the relevant section. The Glossary is helpful, with useful links between definitions and some proofs for the occasions when, if they were introduced in the main text, they would interrupt the thrust of the argument.

The tutorial begins with 'A number game', which involves investigating the remainders when  $n$  and  $n^p$  are divided by  $p$ . When are these the same? This puzzle provides a simple, fun start and is quite intriguing but there is no

attempt to make it relevant to the Theory of Groups until the section on Lagrange's Theorem is reached, well through the tutorial. The problem is abandoned completely as we go into the second section, 'Some examples'. My students enjoyed the game and found it an easy, encouraging opening but, having still not fully mastered the navigation system, initially missed the second, discussion stage.

'Some examples' moves straight into an introduction to permutations and the associated notation. The student is given two examples involving  $(1\ 2\ 3)$  and asked to find others so that s/he is immediately

engaged and made to think rather than just being a passive listener / reader. The use of colour to aid the explanation enhances the clarity and the interactivity, moving objects around the screen, is fun and encourages involvement. Once the preliminary ideas have been covered, the screen is divided so that the white left-hand side becomes a working space, while the grey right-hand side provides more formal proofs, summaries and conclusions. This arrangement is helpful and effective. However the university student (who has recently covered Group Theory) felt that more explanation was needed in the second and subsequent sections as he found them unclear: 'It is not easy to guess what it is talking about'!

The second example (section 4) is the symmetries of the equilateral triangle. In spite of the level of student at which this is aimed, the commentary consistently refers to a 'flip' about the vertical instead of a 'reflection', a term which many children learn at primary school or at least by GCSE. The use of 'vertical' where no terrestrial orientation has been defined is also irritating (since it is a

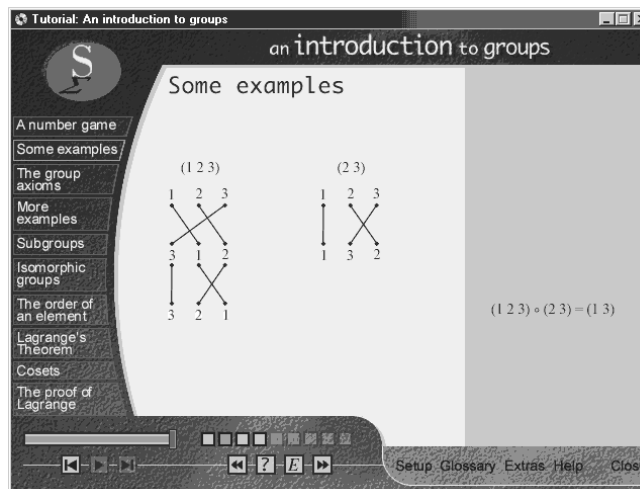


Fig 1 Permutations

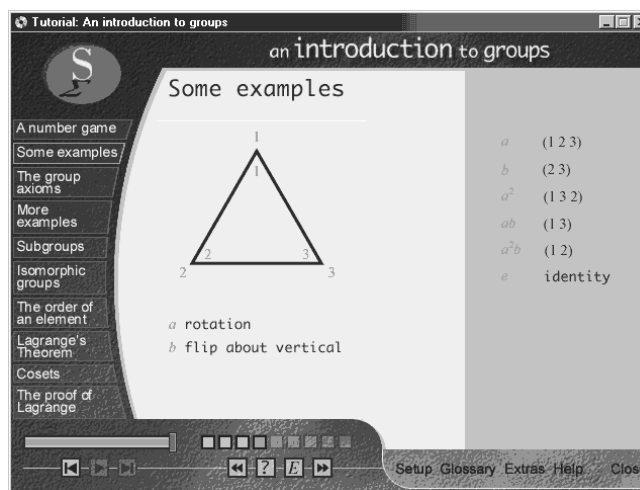


Fig 2 Symmetries of the Triangle

misuse that we spend time trying to persuade students to avoid! My dictionary defines vertical as ‘perpendicular to the plane of the horizon’). In the same way, rotations ‘backwards’ and ‘forwards’ are referred to. The presentation of the symmetries and their combinations is well done and, together with the work on permutations, provides and useful stepping-stone into the later theory. The exercises develop these ideas further and are an essential part of the learning process.

The section on ‘The group axioms’ gives a clear account with straightforward examples and provides a firm foundation on which to build. The students found the definitions good and the ‘game’ in which they could ‘play around’ with the group axioms, very conducive to learning them. However the ‘Exercises’ introduce a problem which occurs with several of the sets of exercises throughout the tutorial: the difference between the level of difficulty of the examples in the ‘text’ and those in the exercises is too great. One expects the questions set to develop the subject and stretch the student and this they do. However there is no gentle lead in with one or two easy questions to begin with. In this section, the first question is on arithmetic modulo 10. At no stage before this has the student met the concept and for some it will be necessary to break off and go into the Glossary. (The topic is actually briefly discussed on the last page of the following section but it would be much better if it were given as a prerequisite for the tutorial, to avoid any interruption in the flow of the main idea.) The method of solution given in the Answers to question one is fairly sophisticated and not at all suitable for the very first problem on this topic. The answers actually say that ‘testing every combination would be possible but tedious’, whereas drawing up a

group table of the given set of four elements is in fact quick and easy and provides a much clearer way of checking the various group axioms. My A level student was so discouraged by this question that she gave the exercise up instead of going on to the very much more straightforward question 3.

In contrast the next section, ‘More examples’, provides a clear and systematic presentation of the groups of symmetries of the regular polyhedra and the relationship of these Dihedral groups to the symmetric groups (introduced through permutations). A brief account of Alternating

groups is included. However at this stage, the unsatisfactory use of language again becomes an issue. When dealing with  $A_4$ , the symmetries of a regular tetrahedron are described. The speaker starts by talking about ‘rotating a face while keeping a vertex fixed’. This is clear but the phrase quickly changes into ‘rotating about a face’, which is taken to be the same thing! The exercises for this section are much better structured than for the previous section.

‘Subgroups’ follows, with good exposition, the use of group tables and opportunities for the student to try his hand at picking out subgroups. The exercises expand the topic and the answers are helpful in developing

systematic approaches to finding subgroups. In the interests of precision however, the operation to be used in defining the group  $R(x)$  of polynomials should be given as, when it was introduced earlier,  $R(x)$  was simply defined as a set.

Using the cyclic and dihedral groups introduced before, provides a direct way into the definition of an isomorphism. The phrase

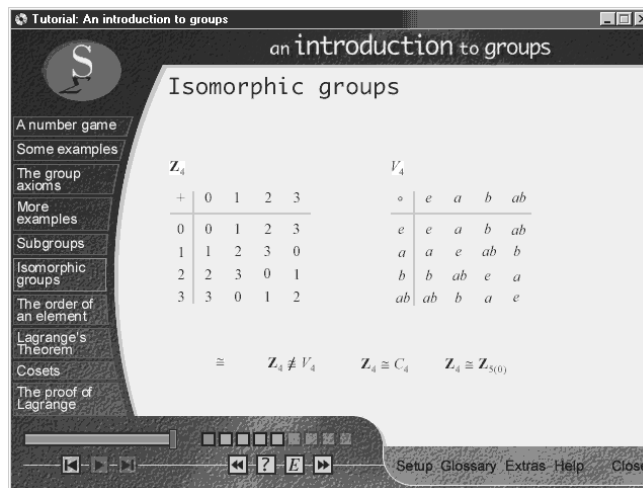


Fig 3 Isomorphisms

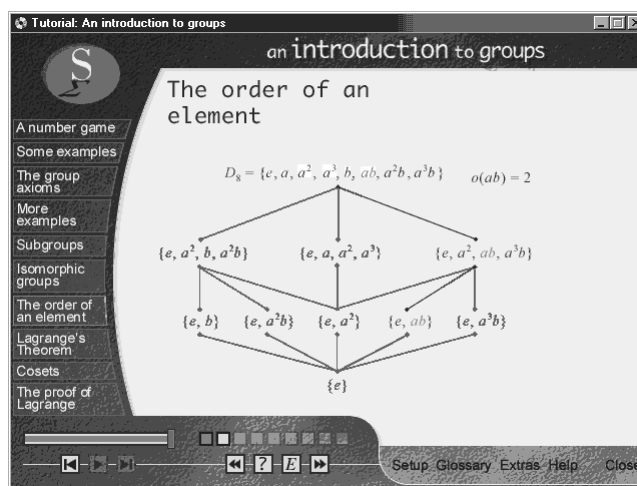


Fig 4 The order of an Element

'pattern matching', though helpful, could perhaps be allowed to evolve into something which places more emphasis on preserving the operation. The student is able to play around with finding isomorphisms between  $C_6$  and  $Z_{7(0)}$  but there is a danger that this will just be by trial and error as no clues are given as to how to ensure that the mapping chosen will be an isomorphism (for instance by mapping self-inverse elements to self-inverse ones). The treatment is reasonably rigorous. A result observed from the group table, such as  $D_6$  not  $\cong C_6$ , is followed up with a more formal proof. Further formal proofs are provided in the answers to the exercises but, again, it would be helpful if more could have been given and explained in the 'text' through a worked example. As in earlier cases, following the tutorial's quite simple explanations, the exercises tend to throw the student in at the deep end!

The CD progresses through a clear development of the Order of an Element, with good use of colour to highlight important words and definitions, supported by proofs in the Glossary. The ideas need careful thought and the students found they had to follow the explanation on the second page through several times but they did then understand it. Lagrange's Theorem itself is introduced through straightforward and interactive examples. Before reaching the section of proof, Cosets are clearly explained with more animated examples which the students enjoyed doing (although a gremlin seems to prevent the smooth running of part of this). The exercises provide the necessary further practice and, in this case, are pitched at an appropriate level.

Finally we reach the proof of Lagrange's Theorem. In order to complete this, equivalence relations and the idea of 'partition' are introduced. This last section goes quite quickly and has no exercises. Although the proof is completed, the fact that the crucial statement

$$|G| = [G : H] \times |H|$$

follows from the previous results, is not given enough explanation, leaving the novice worried at this final hurdle.

## Conclusion

This CD is well presented and the students enjoyed using it. They found it a generally clear and useful introduction to the subject so that, when the A level student covered the topic later in her course, she was able to read through the textbook quite quickly and said that the CD had most definitely helped her understanding of the concepts and methods involved. The interactive examples lighten the work and the voice is well modulated and reassuring. The combination of the chatty, informal approach and the more rigorous presentation in the right-hand column, provide a good balance and through this, together with the glossary and the solutions to the exercises, the student is introduced to several useful techniques of proof. Including a few harder examples in the text would be of benefit

and the exercises do need to be supplemented with some 'bridging' examples, leading the student from straightforward questions through to the more demanding ones, while some of the solutions have to be approached with caution as there are errors. The ability to see two pages at once would be extremely useful, for instance a question together with its answer, a question and a glossary definition or a question and the relevant text. There are other difficulties: clicking 'Using the extras' from the Help menu, froze my screen every time!

'The trouble with learning from a CD,' remarked one of my colleagues, 'is that you can't see exactly how far you've come or how much there still is to do. With a book you can hold the whole topic in your hand.' In fact, this CD deals with this matter very well. One can see how many sections there are and, by selecting any section, how many pages it contains. Once mastered, navigation through the tutorial is as easy as flicking backwards or forwards in a book and the ability to rewind by pages in just seconds, means that any paragraph can be repeated as often as required.

This CD is not a complete substitute for the use of a textbook but does provide a very accessible and appealing way into the subject.

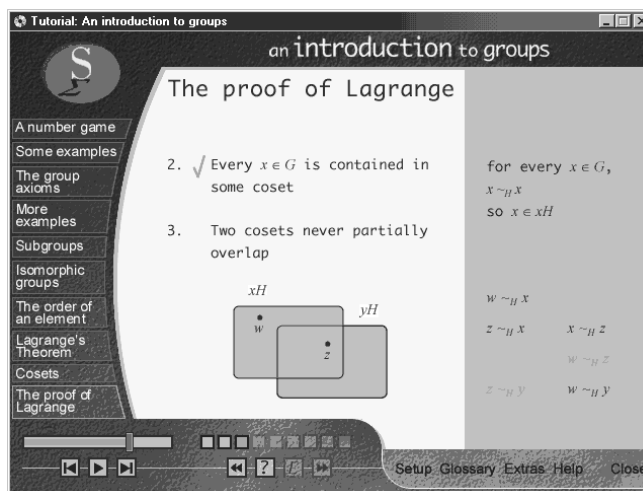


Fig 5 Part of the Final Proof

### Supplier Comments from David Saunders, EBS Trust

This is a generally positive review, and I'm glad that the two student testers found it "a generally clear and useful introduction to the subject". A small instruction card will be included in future copies of the CD, to avoid the short delay in mastering the navigation system that the reviewer mentions. Many of the other remarks made are fair comments. I'm not sure, though, whether I would agree with all the concerns about what the reviewer calls "misuse of language": not even mathematicians use a single word to describe each particular situation, and where everyday ideas are used to introduce a mathematical topic then it's surely reasonable to use the everyday words whenever they don't contradict a precise definition. For instance, the manufacturers of television receivers used to include a knob labelled "vertical hold" without worrying about defining a terrestrial orientation, and everyone knew what they meant.

Overall, the reviewer's final sentence "This CD is not a complete substitute for the use of a textbook but does provide a very accessible and appealing way into the subject" indicates that that the objectives in producing this CD have substantially been achieved, and I hope that many students will be able to benefit from it.

### Response to supplier comments by Jane Bennett-Rees

As far as the use of language is concerned - I stick to my position! Manufacturers of videos may use the term 'vertical' as they wish for the general public but surely for Mathematicians, precise definitions and use of terms go with the territory! Ditto 'flip' - OK for an initial introduction but not to be maintained throughout.

I am glad David Saunders thought I was generally fair.

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