Review of Mathematica 4.1

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Release 4.1 is the latest version of the mathematical software package Mathematica, published by Wolfram Research. At the high end of the market, it competes directly with Maple 6, as an integrated computer algebra system (CAS), and with Matlab 6, in respect of numerical capabilities. Many readers will already know that the other powerful and highly regarded CAS, Macsyma, is no longer commercially available. Earlier versions of Mathematica were reviewed by Daniel Moore [1] and myself [2]. Much has also been written in these quarterly newsletters, over the last decade, about imaginative uses of Mathematica and of related materials in the teaching of mathematics to university and college students. Mathematica is firmly established as a versatile teaching tool in many educational establishments throughout the world, in addition to the extensive use that is made of it in research and development both in universities and industry/business.

I shall concentrate in this review on highlighting the new features which I think have relevance to the teaching/learning applications. Although there are very many improvements and new features in Mathematica 4.1, not all have relevance in the classroom.

The software comes on a CD, and, according to the Getting Started booklet, a full installation under Microsoft Windows occupies about 180 Mb of hard disk space. This includes on-line versions of the Mathematica Book and other documentation. It is worth adding that the supplied versions of the Mathematica Book and the manual for the Standard Add-on Packages are the same as for the earlier version 4.0. It appears that the hard copy manuals are only re-written for major upgrades. I installed the software on a PC with a 650 MHz Pentium III processor and 512 Mb of RAM, running under Windows 2000 – a specification which far exceeds that recommended.

Assumptions

Version 4.0 of Mathematica introduced assumptions to the Simplify and FunctionExpand commands, in the sense that it became possible to specify the domain of a variable. For example, entering Simplify[Sin[n Pi], Element[n, Integers]], produces the expected output. This was a very useful enhancement. The options in Element are Complexes, Reals, Algebraics, Rationals, Integers, Primes and Booleans. If Mathematica cannot decide a domain question, the output echoes the input, but in a form which if interpreted under normal English usage can be misleading. The intention is that the output should be interpreted as a statement whose truth or falsity has not been determined by the program. Thus Element[Sin[Pi/n], Rationals] produces the correct output True or False for n=1, 2, 3, 4 or 6, but in the case of n=5 or n>6, the output has the form "X belongs to Rationals", rather than "Does X belong to Rationals?". A number of further enhancements are given in the Reviewers Guide, but I was unable to find a clear statement of their full extent from the on-line help. Here are some examples.

(a) The double-step function ArcTan[1-x,0]+ArcTan[5-x,0] can be interrogated as to its restriction to an interval of the real axis:

```
Simplify[ArcTan[1-x,0] + ArcTan[5-x,0],x<5 && x>1]
```

has output Pi. In version 4.0, the output is ArcTan[1-x,0], which only provides a partial simplification.

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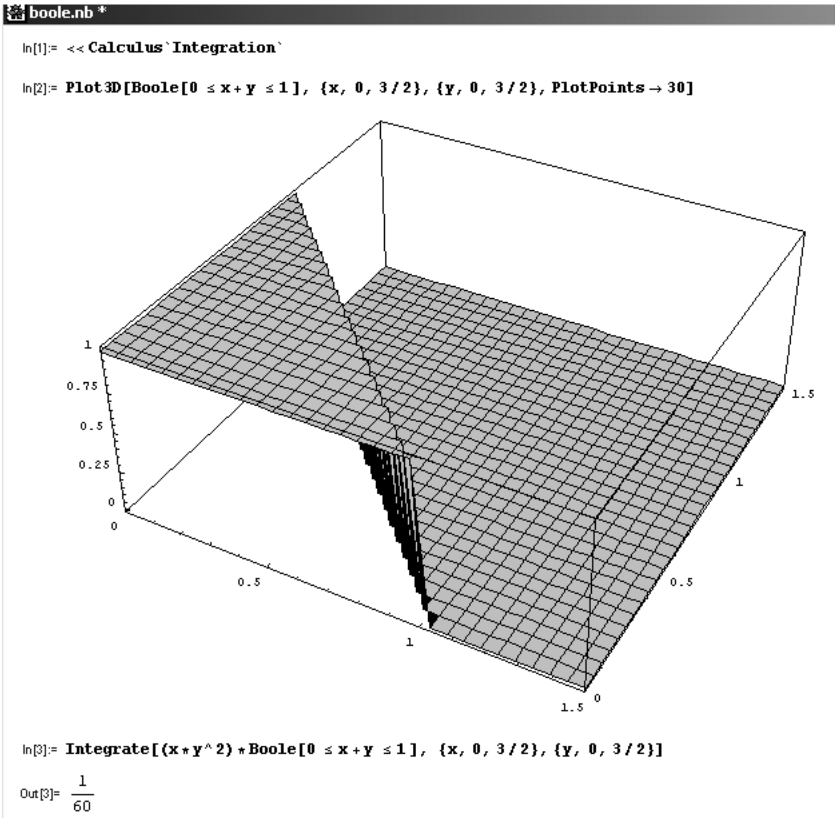

Figure 1: integration over a triangular region

(b) The input

```
Simplify[Implies[a,a]]
```

has output True.

(c) Simplify, with the assumption that n is an integer, shows that the function Csc[(t-n)*Pi]*Sin[(t+n)] has constant value 1.

(d) The input

```
Simplify[Exp[I*n*Pi/2+x],Element[n,Integers]]
```

has output

$$i^n e^x$$

In each of the cases (b), (c) and (d), version 4.0 just echoed the argument of Simplify.

Double Integrals

It has always been possible to perform certain double integrals as repeated integrals with appropriate limits. Often, however, it is more convenient to specify a region of integration implicitly by means of algebraic inequalities. Mathematica 4.1 contains a new package Calculus‘Integration‘ which provides the tools to handle integration over such regions. There is also the capability to deal with piecewise functions. Figure 1 shows the integration of a function over a triangular region – the integral is simple enough that the reader can easily check the answer!

The new package has a help page containing a useful selection of examples.

Sparse Matrices

Sparse matrices are of such importance in applications, for example finite-element methods, that many undergraduate syllabuses in numerical linear algebra now contain some discussion of their theory. In Mathematica 4.0, a sparse matrix solver was introduced in the Developer context. In the latest version further functions have been added to convert between sparse linear systems and sparse matrices. Figure 2 contains the code for generating a sparse linear system, albeit of small dimension, together with its sparse representation and solution.

The SparseLinearSolve function is orders of magnitude faster than LinearSolve when applied to large sparse systems, and utilises very much less memory.

Graphics

One of the major strengths of Mathematica from the very earliest versions has been the quality and versatility of its graphical capabilities. The new package Graphics‘InequalityGraphics‘ provides tools for plotting regions defined by inequalities. The new functions in the package are called InequalityPlot, InequalityPlot3D and ComplexInequalityPlot. Figure 3 shows examples of two of these functions.

These examples just give an inkling of what is possible, with a little experimentation and ingenuity. There are some really quite exotic plots that can be obtained by using the Table function to put together sequences of InequalityPlots.

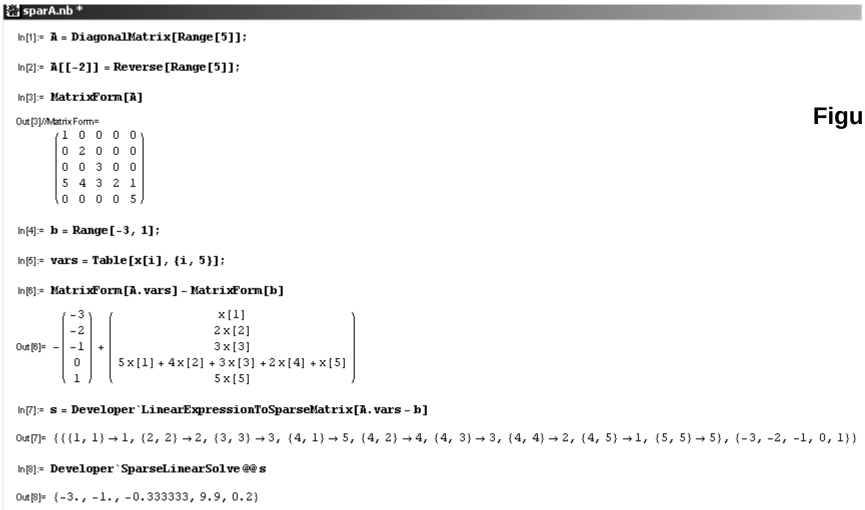

Figure 2: code for a sparse linear system

Other enhancements

There have been further additions to the set of differential equations which can be solve analytically. For example the Riccati equation

```
DSolve[D[y[x],x] = = c*x^m+b*(y[x])^2,y[x],x]
```

now produces an output, albeit rather complicated in terms of Bessel functions. The earlier version of Mathematica could only solve the case where b=1.

Other improvements will probably have less relevance in the classroom, but it is worth mentioning that, for power users, there has been a speed-up in many functions when applied to large datasets. The improvements to MathML integration and other web related issues will be of interest in the research context.

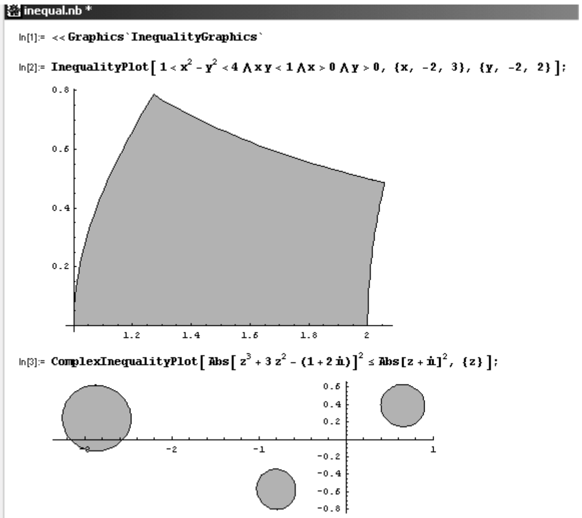

Figure 3: examples of inequality functions

References

[1] D Moore, Review of Mathematica 3.0, *Maths&Stats* Feb 1997

[2] N Backhouse, Review of Mathematica 4, *Maths&Stats* Aug 1999

**Supplier response from Jon McLoone,
Wolfram Research Europe Ltd**

Readers may be interested to know that much of the motivation behind the additional web related features which Dr Backhouse mentions are in preparation for a new technology which we are about to release - webMathematica. This is of direct relevance to teaching applications as it allows users to serve active calculations, tests and courseware over the web powered by Mathematica. More information and examples of its use are available at http://www.wolfram.com/products/webmathematica