# Matching the First Year Mathematics Curriculum to the Student Profile

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Matching the first year maths curriculum to the abilities of incoming students is a difficult task. In any particular topic it is difficult to know what one can assume as a starting point. Here we describe how tests conducted over the last six years on incoming students can be used to provide a rough and ready guide in this [1, 2, 3]. It was found that these results were quite stable from year to year, and between different groups of students with similar grades. There was no evidence of any particular trends, or of any affect on the values due to C2000. So, averaging over the six years, Tables 1 – 4 (see Appendix) give the probable preparedness (pp) values for most of the core A-level topics in Algebra, Functions, Geometry and Trig and Calculus for students with grades A-E and (UK) non-standard A-level equivalent, N (this is a rather heterogeneous group consisting of HND, BTEC, Access, etc). For example, the pp value of 0.3 for partial fractions (Table 1) for grade D students means that if you gave a class of D students a simple partial fractions problem, on average only 30% are likely to get it perfectly correct (or 60% 'half correct', etc!) quickly, efficiently and unaided. In any event you would need to do a considerable amount of revision, and maybe even treat the topic from scratch. Of course, within any particular grade, say B, there will be some variation in performance, but we hope that this is not too great. In general it does little harm to err on the generous side and assume the worst possible case for the pp values.

### *The topics*

The gist of the questions and topics covered is given in the column headings of the tables and is explained more fully in [2]. Within the four blocks the questions are mainly straightforward, one step, standard problems – solve a quadratic, differentiate a polynomial, etc. Some questions are inevitably multi-step, such as integration by partial fractions. But such things are usually taught as a single technique, and we are essentially asking whether they can complete the technique fully or not. In the full tests there are some questions that are really multi-step, or test other higher order skills, but for the purposes of designing a suitable curriculum I have omitted these and relied simply on the basic core facts and skills. It is worth noting however that often such multi-step questions yield low results for all but the best students. The first year curriculum therefore also has to start exercising and developing such higher order skills. Some of the topics in the tables combine results from different variations of the topic – for example 'Use of linear substitution' (Table 4) is averaged over three applications of substitution in integration – however, I hope that the entries give a realistic impression of the students' skills in such topics.

### *The probable preparedness values*

As described in [2], the probable preparedness values are obtained by averaging the performance of students of given prior qualification in Mathematics in a wide and fairly comprehensive range of A-level learning outcomes. The tests designed for this are paper-based, given in the first term of the first year, deliberately without any revision, and with no aids, in a tight time-scale. The results therefore reflect what the students know and can do competently 'off the cuff'. Hopefully it is what one can rely on, in class, amongst the bulk of the students. Sometimes results arose where, for example on average grade E students performed better than grade D in particular topics. I can't explain these, and doubt that they have any practical significance, but I have left them as they are rather than doctor the data. Of course the results are only a crude picture anyway – but hopefully better than nothing! Further details of how Tables 1-4 are arrived at can be found at http://ltsn.mathstore.ac.uk/newsletter/feb2003/matchstudprofile, where you can also find Excel versions of the tables if you want to evaluate pp values for your own intake. Accepting the results given in Tables 1-4 it is possible to design the curriculum to meet the needs of any background of students. I do not believe it will change dramatically in the next few years, but in any case once the basic curriculum is matched to your intake, a brief initial assessment can be used to detect any need for dealing with individual students or overall changes.

### *General observations*

Most providers will have a mix of entry qualifications ranging over A to N, and few institutions will have, for example all grade A or all Non-standard A-level equivalent students. There are however some interesting suggestions regarding results from the individual grades. Inspection of the tables indicates that in practical terms we might conclude the following. There is a wide skills gap between grades A and B, little difference between B and C, a slightly larger gap between C and D which is virtually on a par with E. There is then another large gap between grade E and the N group. Similar breakdowns have been noted by others, from different perspectives [2, 4]. These observations may have some implications for admissions policies. For example, if you are admitting grade B, then with a minor change in curriculum you may be able to accommodate grade C students. If you are taking D, then you might as well take E also. The large gap between E and N suggests that either the threshold for A-level equivalence may need to be raised, or the N have to be taught separately, with significant changes in the curriculum. Of course, there may be other skills and qualities that are more discriminating between the grades and that override the above suggestions.

### *Designing the curriculum*

To illustrate how we can use the Tables to design a curriculum to match any student intake we have given the pp values for two intake profiles at opposite ends of the ability range. Knowing your own intake profile (which does not have to be very accurate) you can easily repeat the exercise and design an appropriate curriculum yourself.

The fine details of what you do in response to any particular pp value is largely up to you, your resources and the point you want the students to reach. As a rough guide you may, for example, adopt an approach along the following lines:

- 0.7-1 no need to cover the topic – perhaps a little revision
- 0.3-0.7 substantial contact time revision with lots of exercises
- 0-0.3 do from scratch

How we use the pp values will also depend on our programme and curriculum objectives. For the purposes of illustration we will assume that the topics discussed are regarded as essential requirements that must be consolidated.

### *University 1*

This might be a typical 'good' university profile with quite a respectable student intake. However, there is still some work to do in terms of what one would, particularly in the past, have regarded as A-level Mathematics. The first thing to note is that even in those topics with high pp there may be more to do than first apparent. Often, students may 'know' a topic, and be able to do a standard problem, but they take too long over it, lack facility, or are not able to apply it. Or, they may use methods they have met before such as Pascal's triangle, which do not extend to more advanced applications. This does not necessarily mean that revision is essential, but it may be worth providing extra practice in such topics. Of course there will be individual student difficulties that may be identified through a short test or coursework, and addressed by extra support.

**Number and algebra**

We can assume that the majority of the students are happy with basic arithmetic and handling fractions and ratios. They can solve a couple of simultaneous linear equations and expand brackets with adequate facility (but not all will recognise terms such as 'polynomial', 'distributive rule', etc). They are fine at factorising quadratics and can usually cope with simple cubics. Such students can combine two rational functions with linear denominators comfortably, although you will have to tell most of them what a 'rational function' actually is, and adding three rational functions will be beyond the reach of many students. They can solve quadratic equations although some will always use the formula, even with equations such as $x^2 – x = 0$. The majority is adept with simple use of indices, but a fair proportion will be confused about dealing with negative numbers and powers. Most of the rest of the basic algebra topics will need some revision however, of greater or lesser degree. So a first year syllabus would need to contain, even for these well-qualified students:

- Some revision on combining simple rational functions and extensions to more complicated forms
- Some revision on definition and use of identities
- Revision of partial fractions, perhaps with extension to repeated factors and quadratic denominators, increasing facility and speed
- Substantial revision or *ab initio* treatment of completing the square, with applications to max and min and possibly deriving the quadratic solution formula
- Binomial theorem virtually from scratch

This implies significant contact time, depending on how much you want to do. It is certainly too much to expect the students to 'mug it up' for themselves.

**Functions**
These students are familiar with the function notation f(x) and general manipulation and combination of functions, but apart from this, what they have met before, they have not retained very well, and substantial amounts of revision are advisable. The pp values speak for themselves, and much of the material needs doing from scratch. A suitable syllabus might be

- Inequalities
- Inverse functions
- Finite series such as the geometric series
- Infinite geometric series
- Infinite binomial series
- Exponential and log functions

Many of these topics are 'supposed' to have been taught in some A-level syllabuses, but for the majority of even well-qualified students they often are not absorbed sufficiently to be retained for later use. All of the above topics are quite conceptually challenging, cannot be rushed, and imply substantial contact hours.

**Geometry and Trig**
The emphasis of the tests here was on consolidating the absolute essentials such as Pythagoras, definitions and properties of the trig functions and on actually using the basic trig identities. Overall we find that even for good students skills in this area are quite weak. They have the basic ideas of Pythagoras, circular measure, trig definitions, and ideas of distance and equation of a line. However, for the rest they are lost without a formula sheet and a substantial proportion cannot use the Pythagorean identity. A suitable syllabus might be

- Pythagoras and its applications (includes Pythagorean identity, equation of a circle, etc)
- Definitions of the trig functions for the general angle, and properties and graphs
- Inverse trig functions
- The compound angle formulae and their applications (eg double angle formulae)

**Calculus**
Usually, students will be continuing their studies in calculus to higher levels and here we are looking for the right combination of topics to ensure as smooth a transition as possible. However we need to take care not to rush on too fast and skimp on the consolidation of the basics. Students at University 1 seem happy with the basic ideas of slopes of curves and the relation to the derivative, as they are with areas and the definite integral. They also have no real problems with the standard derivatives and integrals (even without formulae sheets!). It is when we come to the rules of differentiation and integration that problems start. Most will be competent with the product and chain rules, although many may lack facility and speed, so some brief revision is probably necessary here. In integration substantial revision needs to be done in substitution and integration by parts, and such things as partial fractions and completing the square need to be done from scratch. More importantly in integration some time also needs to be spent on exercise in choosing the method of integration. A suitable syllabus might therefore be:

- Fast and efficient use of rules of differentiation
- Fast, efficient and discriminating use of rules of integration, including substitution and integration by parts
- Applications of calculus

***University 2***

This is at the other end of the ability scale to University 1. It is simpler in this case to describe what can probably be relied upon and then one simply builds on that. As we will see, we are almost teaching A-level over again (but presumably those parts we need) with these students. There are two reasons why this may be so. So-called A-level equivalent non-A-Level students rarely are equivalent, and in the case of D and E students the material they learn is so tenuously absorbed that little remains by the time they reach university.

**Number and algebra**
We can assume that the majority of the students are competent in basic arithmetic of fractions. Most can expand two brackets but may be slow and pedantic and much more so with three or more brackets. They can do very simple factorising but again are likely to be slow. That is about it! All other topics need extensive revision

and much of it *ab initio* treatment. Clearly, the amount of contact time required increases dramatically, and also the endpoint may need to be compromised. Also, variations between students capabilities and individual difficulties are likely to be more difficult to address.

**Functions**

Although these students know what f(x) means, most of the rest of this area needs to be done from scratch. The large number of non-A-level may explain the virtual lack of background in this area. It is the sort of 'pure mathematical' material that they are less likely to cover. Naturally much more time is needed for these students, and usually one cannot reach the level of sophistication that one does with the higher A-level grades. One may also need to adopt a practical and 'mechanical' approach, demonstrating rather than proving results.

**Geometry and Trig**

Apart from Pythagoras, these students need to do all this from scratch. In this it is important not to overload students with lots of identities and results, but to teach them the absolute essentials in depth and thoroughly. It will take a lot of contact time and you won't get so far as with University 1.

**Calculus**

Again, the pp values speak for themselves. These students will need some revision in the trickier standard derivatives, and then the rest of the calculus needs to be done from scratch.

## *Conclusion*

Probable preparedness values of incoming students with different qualifications give some idea of what can be relied on in the basic topics of A-level mathematics in first year teaching. There are limits to what the students can be expected to revise for themselves and the pp values indicate where substantial chunks of time need to be allocated to consolidate their background. This enables the curriculum to be designed to match the students' backgrounds. It may involve some hard choices to devote core teaching time to some of the topics, and this may have implications for subsequent stages of the curriculum, but if it consolidates the foundation on which we can build, then it is worthwhile. There will still be variations between students and individual student difficulties, but hopefully these will be minimized and more easily dealt with by targeted additional support.

## *References*


[1] Appleby J and Cox W (2002), *Transition to higher education* in Effective Learning and Teaching in Mathematics and its Applications, Edited by Peter Kahn & Joe Kyle, Kogan Page, London

[2] Cox W (2000), *Predicting the mathematical preparedness of first-year undergraduates for teaching and learning purposes*, IJMEST **31**, 2, 227-248

[3] Cox W (2001), *On the expectations of the mathematical knowledge of first-year undergraduates*, IJMEST **32**, 6, 847-861

[4] Chandler, S (1997), *A-level mathematics examinations as a fair assessment of the needs of students post GCSE Intermediate and Higher Tiers*, Teaching Mathematics and its Applications **16**, 157-59


## Appendix: Tables 1-4

| TABLE 1 PROBABLE PREPAREDNESS VALUES IN NUMBER AND ALGEBRA | Prior maths qualifications | Adding and subtracting fractions | Ratio: given a:b = 1:6 and b, find a | Two linear simultaneous equations | Expand and simplify (x-2)(x+3) | Expand and simplify ((x-1)^2)(x+1) | Factorize 2x^2 - 3x | Factorize difference of two squares | Factorize quadratic x^2 - x - 2 | Factorize cubic - real roots | Common denominator 2/(x+1) - 3/(x-2) | Common denominator - three fractions | Partial fractions (x+1)/(x-1)(x+3) | Solve identity s+4=A(s+1)+B(s-1) | Solve quadratic x^2 - 5x +6 =0 | Complete the square x^2+x+1 | Binomial theorem for quintic (2+x)^5 | Indices-multiply (3^5)(3^7) | Indices-divide (3^10)/(3^6) | Indices (-1/3)^7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | 1 | 1 | 0.9 | 1 | 0.9 | 1 | 1 | 1 | 0.8 | 0.9 | 0.7 | 0.6 | 0.8 | 0.9 | 0.4 | 0.5 | 0.9 | 1 | 0.7 |
| | B | 0.9 | 0.9 | 0.8 | 0.9 | 0.9 | 1 | 0.9 | 1 | 0.7 | 0.5 | 0.5 | 0.5 | 0.6 | 0.9 | 0.3 | 0.3 | 0.8 | 0.9 | 0.4 |
| | C | 0.9 | 0.8 | 0.8 | 1 | 0.8 | 0.9 | 0.8 | 0.9 | 0.5 | 0.5 | 0.4 | 0.5 | 0.6 | 0.9 | 0.2 | 0.3 | 0.8 | 0.8 | 0.4 |
| | D | 0.8 | 0.7 | 0.7 | 0.9 | 0.8 | 0.7 | 0.8 | 0.9 | 0.4 | 0.3 | 0.2 | 0.3 | 0.5 | 0.7 | 0.2 | 0.2 | 0.7 | 0.8 | 0.3 |
| | E | 0.9 | 0.7 | 0.7 | 0.9 | 0.7 | 0.9 | 0.7 | 1 | 0.4 | 0.4 | 0.2 | 0.2 | 0.6 | 0.7 | 0.1 | 0.2 | 0.6 | 0.7 | 0.2 |
| | N | 0.8 | 0.5 | 0.4 | 0.8 | 0.5 | 0.6 | 0.6 | 0.7 | 0.3 | 0.3 | 0.2 | 0.1 | 0.2 | 0.4 | 0.1 | 0.1 | 0.6 | 0.6 | 0.2 |
| | | | | | | | | | | | | | | | | | | | | |
| **University 1** | 70A, 20B, 10C | 1 | 1 | 0.9 | 1 | 0.9 | 1 | 1 | 1 | 0.8 | 0.8 | 0.6 | 0.6 | 0.7 | 0.9 | 0.4 | 0.4 | 0.9 | 1 | 0.6 |
| **University 2** | 10D, 15E, 75N | 0.8 | 0.6 | 0.5 | 0.8 | 0.6 | 0.7 | 0.6 | 0.8 | 0.3 | 0.3 | 0.2 | 0.1 | 0.3 | 0.5 | 0.1 | 0.1 | 0.6 | 0.6 | 0.2 |

| TABLE 2 PROBABLE PREPAREDNESS VALUES IN FUNCTIONS | Prior maths qualifications | Meaning of f(x) | Composition of two functions | Linear inequality 1/(3x+2)>1 | Inverse function of x/(x-1) | Meaning of sigma notation | Sum finite geometric series | Sum infinite geometric series | Expand infinite binomial (1+3x)^(-2) | Simplify ratio of products of general exponentials | Simplify (e^A)(e^2B)/e^B | Ln 1 | Log to base 2 of 8 | Log to base 3 of 1/27 | Ln of (e^x)(e^y) | 3Lnx + 2Ln(x^3) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | 1 | 0.8 | 0.5 | 0.5 | 0.7 | 0.5 | 0.5 | 0.4 | 0.3 | 0.6 | 0.9 | 0.7 | 0.5 | 0.6 | 0.6 |
| | B | 1 | 0.6 | 0.4 | 0.4 | 0.7 | 0.4 | 0.4 | 0.3 | 0.2 | 0.4 | 0.8 | 0.6 | 0.4 | 0.5 | 0.4 |
| | C | 1 | 0.6 | 0.4 | 0.3 | 0.6 | 0.3 | 0.3 | 0.2 | 0.1 | 0.3 | 0.8 | 0.5 | 0.3 | 0.4 | 0.4 |
| | D | 1 | 0.5 | 0.3 | 0.2 | 0.6 | 0.3 | 0.3 | 0.2 | 0.1 | 0.4 | 0.7 | 0.4 | 0.2 | 0.2 | 0.3 |
| | E | 1 | 0.5 | 0.3 | 0.2 | 0.5 | 0.2 | 0.2 | 0 | 0.1 | 0.5 | 0.6 | 0.4 | 0.1 | 0.3 | 0.2 |
| | N | 0.8 | 0.2 | 0.2 | 0.1 | 0.5 | 0.2 | 0.1 | 0 | 0.1 | 0.3 | 0.4 | 0.4 | 0.3 | 0.2 | 0.2 |
| | | | | | | | | | | | | | | | | |
| **University 1** | 70A, 20B, 10C | 1 | 0.7 | 0.5 | 0.5 | 0.7 | 0.5 | 0.5 | 0.4 | 0.3 | 0.5 | 0.9 | 0.7 | 0.5 | 0.6 | 0.5 |
| **University 2** | 10D, 15E, 75N | 0.9 | 0.3 | 0.2 | 0.1 | 0.5 | 0.2 | 0.1 | 0 | 0.1 | 0.3 | 0.5 | 0.4 | 0.3 | 0.2 | 0.2 |

| TABLE 3 PROBABLE PREPAREDNESS VALUES IN GEOMETRY AND TRIG | Prior maths qualifications | Pythagoras - two longest sides, find third | Degrees to radians | Radians to degrees | Length of arc of a circle | Trig ratios of special angles | Graph of sinusoid | Inverse sine of 1 | Inverse cosine | Pythagorean identity - given cosine find sine | tan(A-B) in terms of tan A and tan B | Double angles - given sin A find cos 2A | Distance between two points | Equation of a line through two points | Equation of a circle with given centre and radius |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | 0.9 | 0.9 | 0.7 | 0.7 | 0.7 | 0.6 | 0.6 | 0.5 | 0.7 | 0.3 | 0.4 | 0.9 | 0.9 | 0.6 |
| | B | 0.8 | 0.9 | 0.4 | 0.5 | 0.6 | 0.4 | 0.4 | 0.2 | 0.5 | 0.2 | 0.1 | 0.9 | 0.8 | 0.3 |
| | C | 0.8 | 0.9 | 0.4 | 0.5 | 0.6 | 0.3 | 0.6 | 0.3 | 0.4 | 0.2 | 0.1 | 0.8 | 0.8 | 0.3 |
| | D | 0.8 | 0.9 | 0.3 | 0.5 | 0.5 | 0.2 | 0.4 | 0.3 | 0.4 | 0.2 | 0.1 | 0.7 | 0.8 | 0.2 |
| | E | 0.8 | 0.8 | 0.2 | 0.3 | 0.5 | 0.2 | 0.4 | 0.2 | 0.4 | 0.1 | 0 | 0.8 | 0.7 | 0 |
| | N | 0.6 | 0.6 | 0.2 | 0.2 | 0.4 | 0.2 | 0.4 | 0.1 | 0.3 | 0.1 | 0.1 | 0.5 | 0.4 | 0.1 |
| | | | | | | | | | | | | | | | |
| **University 1** | 70A, 20B, 10C | 0.9 | 0.9 | 0.6 | 0.6 | 0.7 | 0.5 | 0.6 | 0.4 | 0.6 | 0.3 | 0.3 | 0.9 | 0.9 | 0.5 |
| **University 2** | 10D, 15E, 75N | 0.7 | 0.7 | 0.2 | 0.3 | 0.4 | 0.2 | 0.4 | 0.1 | 0.3 | 0.1 | 0.1 | 0.6 | 0.5 | 0.1 |

| TABLE 4 PROBABLE PREPAREDNESS VALUES IN CALCULUS | Prior maths qualifications | Slope of line 3x + 2y = 1 | Gradient of curve y = x^2 + 1 at x=1 | Differentiate a constant | Differentiate power function | Differentiate sin x | Differentiate exponential | Differentiate ln x | Differentiate polynomial | Differentiate product | Differentiate quotient | Function of a function rule | Stationary points - polynomial | Integrate power (not equal to -1) function | Integrate reciprocal | Integrate sin x | Integrate exponential | Integrate polynomial | Use of linear substitution | Integrate f'(x)f(x) | Integration by parts xsin x | Integrate 1/((x-1)(x+1)) - partial fractions | Evaluate definite integral | Integrate f'(x)/f(x) | Area under curve |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | 0.9 | 1 | 1 | 1 | 0.9 | 1 | 0.9 | 1 | 0.8 | 0.6 | 0.8 | 0.6 | 0.9 | 0.9 | 0.9 | 1 | 1 | 0.6 | 0.3 | 0.5 | 0.3 | 0.9 | 0.4 | 0.9 |
| | B | 0.8 | 0.9 | 0.9 | 0.9 | 0.9 | 1 | 0.9 | 0.9 | 0.5 | 0.4 | 0.5 | 0.6 | 0.8 | 0.8 | 0.8 | 0.9 | 1 | 0.4 | 0.1 | 0.4 | 0.1 | 0.9 | 0.2 | 0.8 |
| | C | 0.8 | 0.9 | 0.8 | 0.8 | 0.8 | 0.9 | 0.8 | 0.9 | 0.4 | 0.4 | 0.4 | 0.5 | 0.8 | 0.7 | 0.8 | 0.9 | 0.9 | 0.3 | 0 | 0.3 | 0.1 | 0.8 | 0.1 | 0.8 |
| | D | 0.7 | 0.9 | 0.8 | 0.7 | 0.8 | 0.9 | 0.8 | 0.9 | 0.3 | 0.2 | 0.2 | 0.4 | 0.8 | 0.6 | 0.8 | 0.8 | 0.8 | 0.2 | 0 | 0.2 | 0 | 0.8 | 0 | 0.7 |
| | E | 0.7 | 0.9 | 0.9 | 0.7 | 0.8 | 0.9 | 0.8 | 0.9 | 0.3 | 0.1 | 0.2 | 0.4 | 0.7 | 0.6 | 0.6 | 0.9 | 0.8 | 0.2 | 0 | 0.1 | 0 | 0.7 | 0 | 0.7 |
| | N | 0.4 | 0.7 | 0.7 | 0.6 | 0.6 | 0.8 | 0.7 | 0.8 | 0.3 | 0.2 | 0.1 | 0.2 | 0.5 | 0.5 | 0.6 | 0.8 | 0.6 | 0.1 | 0 | 0.1 | 0 | 0.5 | 0 | 0.2 |
| | | | | | | | | | | | | | | | | | | | | | | | | | |
| **University 1** | | 0.9 | 1 | 1 | 1 | 0.9 | 1 | 0.9 | 1 | 0.7 | 0.5 | 0.7 | 0.6 | 0.9 | 0.9 | 0.9 | 1 | 1 | 0.5 | 0.2 | 0.5 | 0.2 | 0.9 | 0.3 | 0.9 |
| **University 2** | | 0.2 | 0.8 | 0.7 | 0.6 | 0.7 | 0.8 | 0.7 | 0.8 | 0.3 | 0.2 | 0.1 | 0.3 | 0.6 | 0.5 | 0.6 | 0.8 | 0.7 | 0.1 | 0 | 0.1 | 0 | 0.6 | 0 | 0.3 |