The Math Everywhere Courseware Series— *Calculus&Mathematica*

Phillip Kent
Institute of Education

p.kent@ioe.ac.uk

Mathematica veterans will probably remember the first version of *Calculus&Mathematica*, which was published in a chunky set of spiral-bound volumes by Addison-Wesley in the late '80s (cf. the review in Maths & Stats [1]). Ten years later, Calculus&Mathematica is still going, and in an expanded series of courseware packages; the whole series now goes under the name "Math Everywhere" (www.matheverywhere.com). It is no longer published in printed form by a mainstream publisher, but is being distributed on CD-ROM by Wolfram Research Inc, the creators of Mathematica, who are promoting educational courseware as a "vertical application" of their own product (other computer mathematics software companies, notably Matlab and Mathcad, are actively doing this as well).

The Math Everywhere web site has example materials to download (use the free MathReader notebook viewer from the Wolfram web site if you don't have Mathematica itself), so rather than discuss here the detailed mathematical content of the courseware, I want to consider the principles on which it has been designed, and the ways in which it has been used successfully with students.

The Math Everywhere courseware

Four titles are currently available:

- Calculus&Mathematica
- DifferentialEquations &Mathematica
- Matrices, Geometry &Mathematica
- Vector Calculus &Mathematica

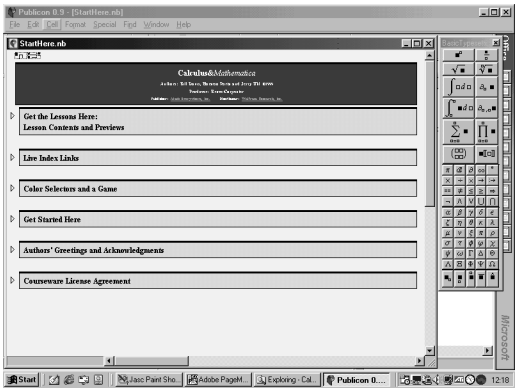

Fig 1 *Calculus&Mathematica* opening screen

Each one comes as a collection of Mathematica "notebook" documents on a single CD-ROM. At the Wolfram Research online store http://store.wolfram.com, the UK price is quoted as £50, but curiously the US price is given as $45. No printed material at all is provided, but the CD does contain all the Mathematica notebooks (complete with the outputs from code examples) in neatly-paginated PDF format for printing. Technically, it's extremely well put together, making good use of the hyperlink feature of Mathematica to link between the different parts, and there is also a full index with hyperlinks. You launch the "Start Here" notebook for each package and this provides access to all the parts; also, you can incorporate the Math Everywhere notebooks within your Mathematica setup, and access everything via Mathematica's Help Browser.

Pedagogical design

Math Everywhere is intended for the North American undergraduate curriculum, having been produced jointly by the Mathematics Departments of the University of Illinois and Ohio State University. It's a product of the Calculus

Supplier's contact details

Wolfram Research Europe Ltd
10 Blenheim Office Park
Lower Road
Long Hanborough
Oxon
OX8 8LN

Tel: 01993 883400
Fax: 01993 883800
Web: www.wolfram.com

Reform movement, and *Calculus&Mathematica* originated in one of the first government-funded pilot projects in the late 1980s. The two principal creators, Jerry Uhl and Bill Davis, are well-known speakers (eg at the annual ICTCM conferences [2]) on their approach to teaching and learning mathematics (both diverted themselves in mid-life from distinguished careers as research mathematicians to work on a teaching project—this has been a notable feature of the Calculus Reform movement as a whole, and something which, to my knowledge, has no parallel in the United Kingdom). Here, for example, is Jerry Uhl on the unworkableness of teaching mathematics by delivering a course of "long lectures" [3]:

> *I was a very popular lecturer and recently won [an] award for distinguished teaching based in no small part on the lecture courses I gave at Illinois between 1968 and 1988. But for the last ten years, I have completely abandoned the long lecture method. Simply put, today's students do not get much out of long lectures, no matter how well they are constructed. The material comes too fast and does not sink in well. The students of the past responded by becoming quiet scribes. Today's students demand more action and accountability. That's why many students cut class and even when they come they often ask hostile questions such as "What's this stuff good for?" … One of the first to note that the lecture system needed to be replaced was Ralph Boas in 1980: "As a means of instruction, lectures ought to have become obsolete when the printing press was invented. We had a second chance when the Xerox machine was invented, but we muffed it." Many math instructors are trying to teach today's students using only yesterday's tools and approaches. And neither the instructors nor the students are pleased with the results.… A few strategically timed and strategically placed short followup lectures (sound bites) can be very effective. But the problem with introductory lectures is that they are full of words that have not yet taken on meaning and full of answers to questions not yet asked by the students.*

So, how does Math Everywhere respond to the problem?

> *Today my calculus, differential equations and linear algebra students get the experience they need through Mathematica-based courseware … The basic ideas are laid out in interactive Mathematica Notebooks in which new issues arise visually through interactive computer graphics. With this courseware, limitless examples are possible almost instantly. If the student doesn't get the point right away, then the student can rerun with a new example of the student's own choosing. … They see for themselves what the issues are before the words go on and generalizations are made. One of our favorite techniques is to give a revealing plot and ask the students to write up a description of what they are seeing and to explain why they see it. In these courses, conceptual questions are the rule and students answer them. Contrast this with the typical student problems assigned in traditionally taught mathematics courses. [3]*

The pedagogical design of Math Everywhere is extremely well-honed, and actually has not changed much since the early versions of Calculus&Mathematica. Each topic is divided into a set of lessons, and there are four notebooks for each lesson—Basics, Tutorials, Give It a Try and Literacy. This is a description of how the lessons are used in courses at Ohio State:

> *When a student is assigned a new homework, rather than listening to a lecture, the students go directly to the computer to begin the Basics. The[se] have carefully explained examples of each concept that the lesson is teaching. This is where a lot of the new vocabulary is introduced. After finishing it, the students goes on to the Tutorials, [where] there are more in-depth examples, and the student is encouraged to interact with the examples. … When the student feels they understand the concepts, they move on to the Give It a Try. The problems are similar to those in the Basics and Tutorials, but in this case, the answers are not given. The instructor assigns certain problems from the Give It a Try section for individual and/or group homework. The student and/or group completes the assigned problems using the Basics and Tutorials as a reference and by interacting with other students, the lab assistants, and the lab instructor. After finishing the electronic lessons, students will then do a Literacy Sheet for the lesson. The[se] are designed to test the students' understanding of the lesson and their ability to do the problems by hand. [4]*

The "How to Use" sections on the Math Everywhere CDs also suggest to use a less sequential way of working through each lesson (which is appropriate for a standalone product), advocating the user to develop a personal "learning style" that works for him or her. Of course, successful learners know this, but it's refreshing to see such advice clearly spelt out for learners who may have had less success in the past—and when it comes to maths courses, there are many of these.

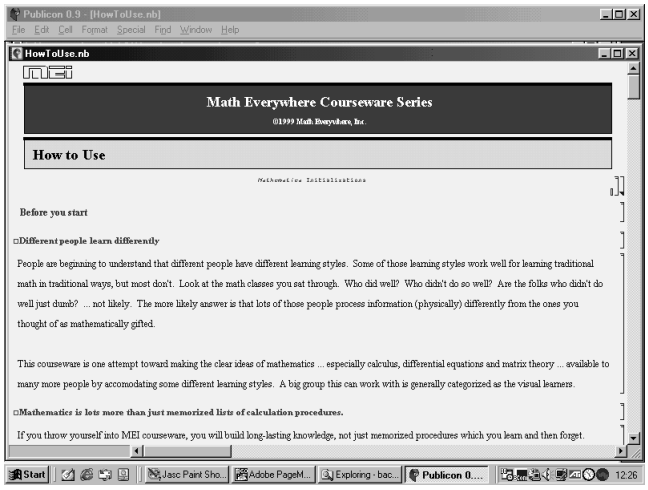

Fig 2 "How to Use" advocates personal learner style

Courseware and courses

If you're going to judge Math Everywhere as courseware, then you need to see how it used in courses at its home institutions at Illinois and Ohio State [4, 5]. They are fully laboratory-based, with the labs being open long hours and staffed by teaching assistants who are senior undergraduates who took the courses several years earlier—so the students get sympathetic advice and the assistants valuable teaching experience. There is a strong collegiate atmosphere (which I can vouch for from several visits to Illinois myself), and collaborative work, with plenty of mathematical discussion, is demanded from the students. Both institutions have done studies to ascertain how well students taking their courses compare in traditional pen-and-paper tests with students from traditional courses, with very favourable results. The student feedback comments reported on the web sites are extremely impressive. There's no doubt that Math Everywhere works very well in its home locations; and they also run successful distance-learning courses, which allow high school students to take introductory calculus courses before they enter university.

Courseware development has been advocated in the past as a primary route to technological innovation in university teaching and learning: you pay "expert" academics to develop a set of teaching materials (based on, say, computer algebra, multimedia authoring tools, or web pages and Java), which materials can then be picked up and used in innovative teaching by "ordinary" academics. For example, the Teaching and Learning Technology Programme in the UK was founded on this principle, and, as we know, the success rate of projects within the Programme was distinctly patchy. In the USA, likewise, the Calculus Reform programme was funded for quite a while according to the same principle: a small number of "expert" institutions were supposed to lead the way, and provide the fruits of their labour for others to follow. It hasn't worked out that way, and the Calculus Reform debate amongst US mathematicians remains highly-charged.

Where Math Everywhere has succeeded, I'm certain that it is because of a marriage between technological innovation, deep-seated changes to teaching methods and assessment (as already mentioned) and developing new approaches to the mathematics. In differential equations, for example, Math Everywhere breaks away from the traditional algebra-oriented approach, with its emphasis on the restricted classes of differential equations that happen to be soluble algebraically, and instead invites students to visualise many examples of flows and trajectories. In pre-computer days, such visualisation techniques were an advanced topic, in that you had to understand a lot about differential equations before you could understand how to construct the visualisations. The brilliant thing that computer software makes possible is that the visualisations (constructed by the software) become a way to understand the basic meanings of differential equations.

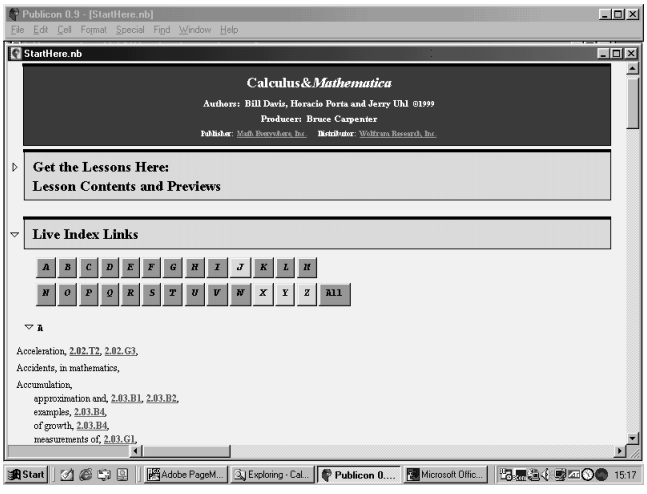

Fig 3 Alphabetic list to links to find specific items

Approaches to Mathematica

One thing you will notice right away about Math Everywhere is that Mathematica is presented to students in a raw, uncensored form. If, say, a non-standard graphical construction is required to illustrate something it is given in full without any attempt to shield the details; here's an example of a piece of input code from the first lesson of Vector Calculus:

```
x = {1.5, 5.3, 2.0};
spacer = 0.2;
h=1;
threedims = Axes3D[h,spacer];
Show[Arrow[X, VectorColor-
>Red],threedims,
  PlotRange->All,ViewPoint->CMView,
Boxed->False]
```

And yet, a knowledge of Mathematica is not specified as a prerequisite for using any of the Math Everywhere packages.

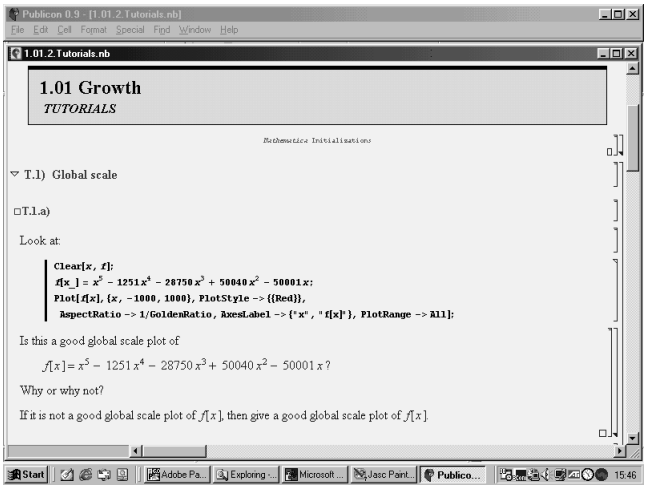

Fig 4 Mathematica code presented in raw form

Now Mathematica has a reputation for being difficult to use, and my own experience teaching Mathematica to undergraduates confirmed that, with very many students struggling persistently with the intricacies of the syntax. (And I know that it was particularly a Mathematica problem, because courses based on Maple, with almost identical content and taught by the same people, were radically different in terms of student satisfaction and engagement.)

It is this kind of difficulty with Mathematica that has led many people to want to put a "more accessible" layer on top of Mathematica, which range from programming special commands that hide some of the complexities of syntax, to building alternative interfaces which (partially) replace the typing of Mathematica commands (see my earlier review of some products which attempt to do that [6]). Remarkably, Math Everywhere sails blithely against these problems. How so? This is where overall course design is crucial. Math Everywhere works in its home institutions because the entire course—including some portion of the final assessment—is based on Mathematica, and there is a collegiate, supportive atmosphere in which the teachers, teaching assistants and students are all in it together. In such a context, the learning of the software itself is simply part of the overall scheme of things. But, the flip-side of this is that if you want to use Math Everywhere outside of such a context then you will need some prior experience with Mathematica.

Conclusion

I have offered an upbeat review of Math Everywhere, and this reflects my biases for Mathematica in particular, and computer algebra software in general. However, what's notable about Math Everywhere, beyond its use of computer algebra, is the entire approach to innovative mathematics teaching and learning. It is an example that really ought to be better known.

Notes

[1] Jon Greenman, *Maths & Stats* vol. 4, no. 3, 1993.
[2] The International Conference on Technology in Collegiate Mathematics is held every November in a different US city, and is an excellent gauge of the state of technology in undergraduate maths teaching in the USA. See http://www.awlonline.com/ictcm for the forthcoming conference, and http://archives.math.utk.edu/ICTCM for proceedings of past conferences.
[3] Jerry Uhl, "Why (and how) I teach without long lectures", http://www.vpaa.uillinois.edu/tid/resources/uhl.html
[4] Calculus&Mathematica at Ohio State University: http://socrates.math.ohio-state.edu
[5] Calculus&Mathematica at the University of Illinois: http://www-cm.math.uiuc.edu
[6] "Educational add-ons to Mathematica: Joy of Mathematica, Calculus Wiz and MATeacher", *Maths, Stats & OR*, vol. 1, no. 3, August 2000.