Something that worked for me...

Title: **Notes On Making Mathematical Notes For Your Course** *Author:* **C T J Dodson, UMIST**
Email: **dodson@umist.ac.uk**

About thirty years ago, I produced a handwritten summary of guidance notes for students on mathematical language, symbols, logic and proofs. I found this document again recently and decided that it was still worth distributing, at least to mathematics majors and joint majors since it does seem to capture some essential skills that a student should acquire during a first semester at university. The typed document is available in PDF format at http://www.ma.umist.ac.uk/kd/ma351/notes.pdf and is reproduced here. It is self-explanatory.

Language, symbols, logic and proofs

1. *Language*

(i) Make sense to a computer (*not* a telepath) using same axioms.
(ii) Punctuate into sentences and paragraphs. Brackets are useful.
(iii) Signpost arguments: before, during and after the development.

2. *Symbols*

(i) Declare the sets in use; eg

$A = \{x \in R^m \mid (\textit{statement about } x))\}$

(ii) Introduce an element by declaring its origin and make clear how it was chosen or if arbitrary, from which set.
(iii) Use '=' as an abbreviation of aequare, `to equal' (Descartes introduced it as a in the 17th century). It can be used only between sets, or between elements of one set; not as an abbreviation of 'is'.
(iv) Use ≤ only between real numbers (or elements of posets, later).
(v) Use ⇒ only between statements; not to begin a sentence.
(vi) Use brackets for clarity, eg to separate quantified phrases:

$(\forall \varepsilon > 0)(\exists \partial > 0) : |x - a| < \partial \Rightarrow |f(x) - f(a)| < \varepsilon$

3. *Logic*

(i) **If . . . , then . . . .** (Follow 'if' with 'then')
(ii) Learn how to read and negate (¬) statements:

$(P \Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$

Try these statements in words and symbols; note the way that 'and' and 'or' arise as mutual negations:

Statement S_1: All lecturers are hairy **or** have a glass eye. Symbolically $S_1 : (\forall x \in L) \quad H(x)$ **or** $G(x)$

Negation $\neg S_1 : (\exists x \in L) : \neg H(x)$ **and** $\neg G(x)$

Note the difference between S_1 and the following S_2:

Statement S_2: All lecturers are hairy **or** all lecturers have a glass eye.

Symbolically $S_2 : (\forall x \in L, H(x))$ **or** $(\forall x \in L, G(x))$

Negation $\neg S_2 : (\exists x \in L : \neg H(x))$ **and** $(\exists y \in L : \neg G(y))$

4. *Proofs*

(i) A common error is to assume the result and show it is reasonable.
(ii) Clearly declare definitions you need.
(iii) Know a standard layout for each of these seven situations;

(1) $(P \Rightarrow Q)$:-
Either, directly
assume P is true and then deduce that Q must be true.
Or use the equivalence $(P \Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$
assume Q is false and then deduce that P must be false.

(2) $P \Leftrightarrow Q.$: - (a) $P \Rightarrow Q$ **and** (b) $Q \Rightarrow P$

(3) $A = B.$: - (a) $A \subseteq B$ **and** (b) $B \subseteq A$

(4) $(\forall x \in L))Q(x).$: - Take **arbitrary** $x \in L$, ... deduce $Q(x)$

(5) $(\exists x \in L) : Q(x).$: - Find **any** $x \in L$, with $Q(x)$ true

(6) $(\exists! x \in L) : Q(x).$: - Do (5).
Suppose also x^1 satisfies (5). Show uniqueness of x by proving $x=x^1$.

(7) Induction :- Show P_1 true. Assume P_k true for an **arbitrary** k ≥ 1. Show P_{k+1} true.

Making lecture notes useful

1. *Definitions and theorems*

(i) Augment your notes with examples, and especially with crucial non-examples. You are expected to find these for yourself as well, eg in books and tutorials.
(ii) Definitions often arise out of crucial non-examples, and the obscure parts of proofs are frequently needed to circumvent non-obvious pathological cases. Find out which examples broke the teeth of previous theories and why.
(iii) Some definitions are 'natural', for example: a metric space and a dual vector space. Find out why.
(iv) An index for your notes is worth having.

2. *Tutorial problems*

(i) Read your (augmented) lecture notes around the definitions and check other sources (more than one book).

(ii) Decide which layout under 4(iii) above that you need.
(iii) If difficult to begin, then first try a simplified problem.
(iv) Most problems appear as examples or theorems in some book.
(v) Consult your tutor if a reasonable attempt fails.

3 Revision
(i) Reading proofs is only for insomniacs
(ii) For revision, write out illustrative examples in parallel with the proofs. Contrast non-examples.
(iii) Use a range of books not just one. Use your tutors.

Title: **NPV and IRR using Excel**

Author: **Patricia Wackrill, Middlesex University**
Email: **p.wackrill@mdx.ac.uk**

Our students meet the idea of NPV and IRR in year 1 of their course. The train of thought which led me to use Excel in this context started with a workshop put on by the Operational Research Society on "Helping academics to meet the changing needs of employers" in March last year. The strong message I came away with was that employers needed experience and skill in using spreadsheets.

Now, of course I have to teach students to understand and compute the Discount Factor using a calculator but Excel provides an opportunity to experiment. Different discount rates are a simple starting point. Then suppose that two different projects have the same total net inflow over a period of years, but that one has to wait a longer time to receive one of the bigger inflows. Once one has set up the formulae to compute the Discount Factors and the NPV for each year of a project, which is a useful skill in its own right, one can vary the discount rate or shuffle round the cash flows, and see the NPV getting lower when one has to wait longer for the biggest cash flows. Using Excel as a "What if?" tool is a useful advance on just using it to work out complicated bills.

The three projects with the same total net inflow which I used were:

Year	Project A Net cash inflow £	Project B Net cash inflow £	Project C Net cash inflow £
0	-20000	-20000	-20000
1	4000	4000	16000
2	16000	10000	10000
3	10000	16000	4000
Total	10000	10000	10000

Excel's chart facility can be used to give a visual impression of the different cash flows.

Whatever one's opinion on whether IRR is a useful measure or not, students should know what it is. The idea can be very well illustrated by pumping up the discount rate, until the NPV goes negative. One can then home in on two consecutive integer rates between which the NPV turns negative. Having discovered that the NPV turns negative between rate r% and rate (r+1)%, one can then remind the students of similar triangles, and teach them about linear interpolation. Once again the calculations could be tedious, but one can discuss how Excel can be made to find where the straight line joining the point representing positive NPV with rate r% to the point representing negative NPV with rate (r+1)% cuts the rate axis. I repeated my block for each project so that one copy could show the rate r% with the positive NPV (=NPV1) and the second copy could show the rate (r+1)% with negative NPV (= NPV2).

The formula for the IRR was then:

$$r + NPV1/(NPV1 - NPV2)$$

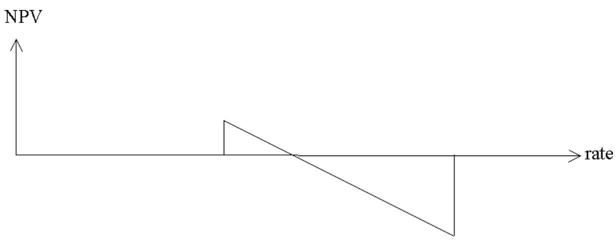

Figure 1. Linear interpolation

Editors Note...

We would like to thank Patricia Wackrill for allowing a copy of the spreadsheet file to be made available via our website at:
http://ltsn.mathstore.ac.uk/newsletter/aug2002/pw/npvirr.xls