

Leibniz: A word processor with mathematical intelligence?

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Downloadable from <http://www.leibnizsoftware.com>

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Requirements: needs Mathematica 3.0 or greater, or access to a server version of Mathematica leased by the vendor.

Cost: site licenses are \$10 per machine with a minimum of 5. An unlimited site license is available for \$1000. The site license version is equivalent to the full 'long form code' version that normally sells for \$30 a copy.

Leibniz is a simple and straightforward word processor, with a very big difference! Within the text it is possible to create mathematical expressions using templates, select them and then pass them directly for evaluation to an associated Mathematica Kernel, via MathLink. The results are then pasted back into the Leibniz document.

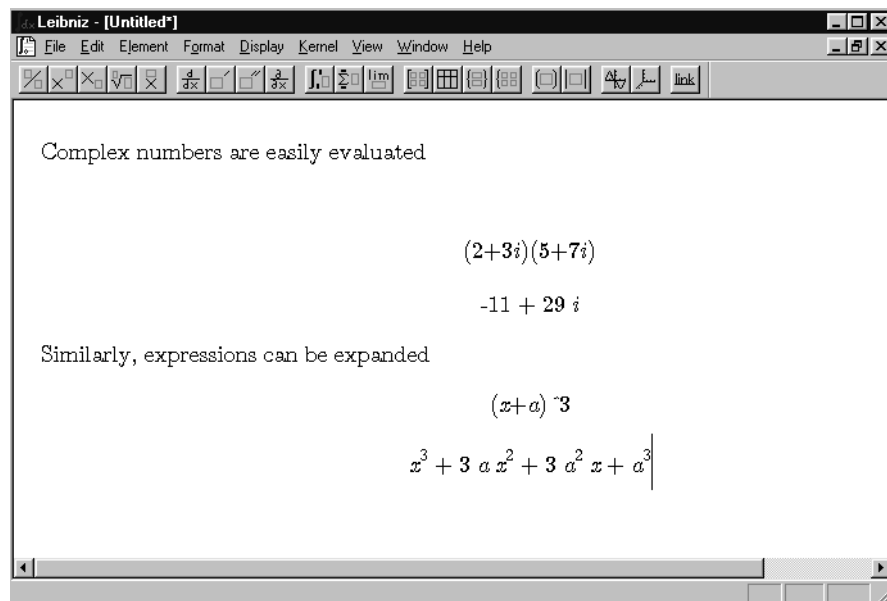


Fig 1: A simple Leibniz document

Leibniz has been developed by Joe Gregg, who is an associate professor of Math and Computer Science at Lawrence University in Appleton, Wisconsin. Professor Gregg has been working with Mathematica for close to a decade, and started developing Leibniz in 1993 in an effort to make Mathematica more accessible for students and teachers alike. It can be used to teach courses in calculus, differential equations, and linear algebra. This review lists some of the features, and further details can be found at <http://www.leibnizsoftware.com/about/Newest.html>

There are two possible modes of operation:

- Using local copy of Mathematica (version 3.0 or later)
- Using a server version of Mathematica operated by Leibniz Software, Inc. Unfortunately, due to licensing restrictions imposed by Wolfram Research, not all of the features of Leibniz will be available when you use the calculation server

Leibniz is supplied as a self-installing archive and comes with a variety of documentation that gives more than enough information to get started and learn the basics. The package includes documentation on installation, getting started and some examples of how Leibniz works with simple algebra, calculus and linear algebra.

Introduction

Leibniz has two methods of manipulating expressions.

Direct Evaluation: To evaluate $(2 + 3i)(5 + 7i)$ or expand the expression $(x + a)^3$, all you need to do is select it, hold down the shift key and press enter. Leibniz will send the expression to Mathematica for calculation and place the result as shown in Fig 1. It's as simple as that, and no syntax to learn!

Drag and drop: To do a drag and drop calculation in Leibniz, select the term you want to move and drag it. As you do so, Leibniz will give you feedback showing you the target of the drag and drop operation. For example, given the equation,

$$x + a = b$$

You can simply drag the a to the right-hand side to get

$$x = -a + b$$

Or, given $(x^2(1+x))/(x^n(1-x))$, grab the x^2 term in the numerator and drag it on top of the x^n term in the denominator, and Leibniz gives

$$\frac{x + 1}{x^{n-2}(-x + 1)}$$

More examples of the use of Leibniz

Solving Equations: Leibniz has two basic techniques for solving equations. To solve the equation

$$x^2 - 3x + 2 = 0$$

for the variable x , drop it on the expression

$$x = ?$$

Or select x , right click, and select the option "solve". In both cases, Leibniz returns $x = 1$ or $x = 2$. Similar techniques are available for systems of equations. Leibniz seems to handle all the standard mathematical notation that might be encountered in a first course on calculus or differential equations.

Working with inequalities: Leibniz manipulates inequalities in a very simple intuitive way, by dragging things from one side of an inequality to another. For example, given that

$$-4 < 3 - 2x < 12$$

it is possible to select and drag the 3 to get

$$-7 < -2x < 9$$

and then drag the factor -2 to get

$$\frac{7}{2} > x > -\frac{9}{2}$$

Matrices, Determinants and Linear Algebra: Matrices, and many other mathematical constructs, are entered via a simple template shown on the menu bar in Fig 1. To create a determinant, select the matrix and then the absolute value button in the toolbar. To evaluate the determinant, select it and shift-enter. To compute the transpose of a matrix or vector, put an exponent of T on it, select the whole construction, and shift-enter. Similarly, to compute a matrix inverse, give the matrix an exponent of -1, select the whole construction, and shift-enter. You can also compute matrix inverses by dragging and dropping. Leibniz solves systems of linear equations using Gaussian elimination, either proceeding directly to the solution or allowing the user to carry out the row reduction and back substitution.

Fig 2 shows an example of how Leibniz can be used to find the eigenvalues of a matrix. Alternatively, Leibniz can do the work for you by passing the appropriate Mathematica command direct to the kernel for evaluation.

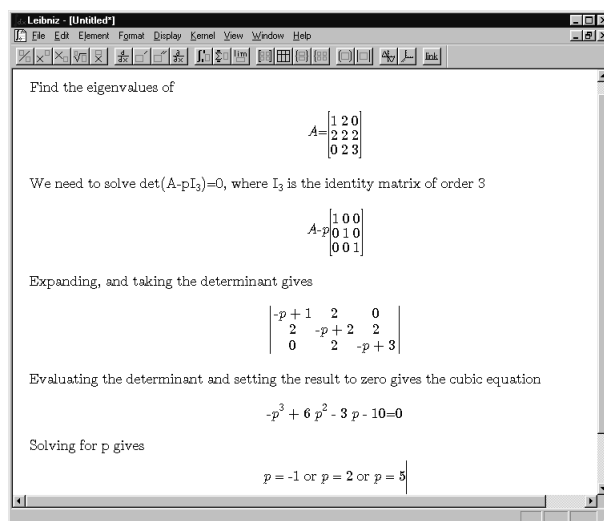


Fig 2: A Leibniz worksheet finding eigenvalues

Calculus: Leibniz gives you the option of working with either one of the two traditional notation styles used with derivatives. For example, to differentiate $\sin(x^2)$ the templates can be used to build up the expression

$$\frac{d}{dx} \sin(x^2)$$

Alternatively, the third derivative can be found using the notation $(\sin(x^2))^{(3)}$. To evaluate it select and click!

Leibniz offers a variety of techniques for working with integrals. Some of these techniques allow you to manipulate an integral as a symbolic construct, while others are geared toward computing integrals. For example, to evaluate the integral

$$\int \frac{x}{1+x^2} dx$$

using the substitution $u = 1 + x^2$, select the substitution and drag it onto the integral. The result can then be simplified as shown in Fig 3 by dragging the factor $\frac{1}{2}$ through the integral to tidy up. Notice that Leibniz automatically adds a constant of integration. To return to the original variable, drag the substitution onto a standard integral.

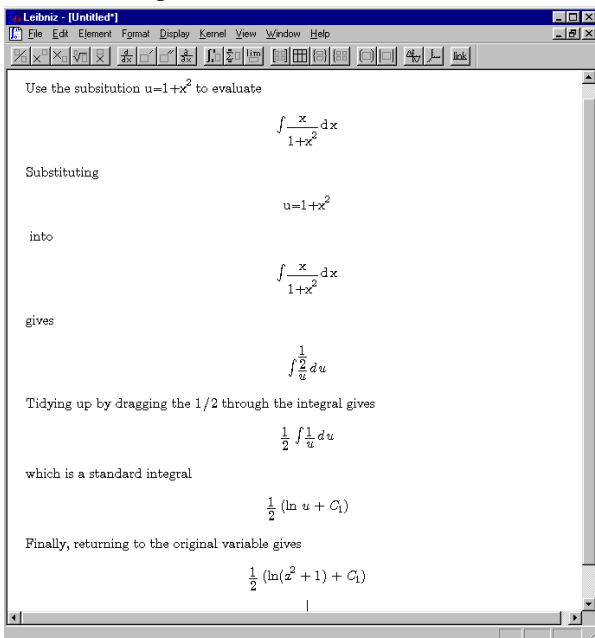


Fig 3: Using Leibniz to evaluate an integral

Leibniz can also be used to guide students through the process of integration by parts, see Fig 4.

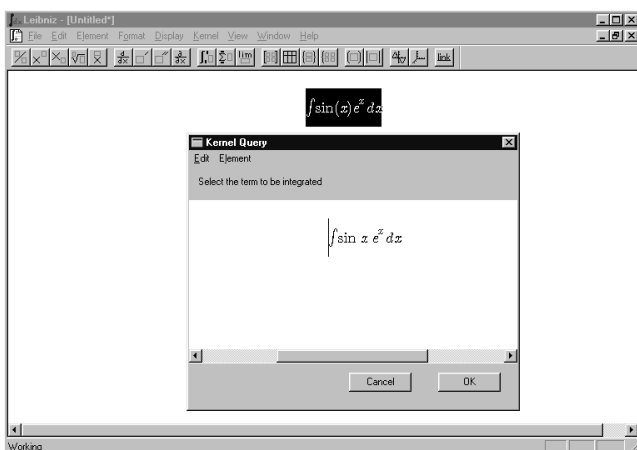


Fig 4: Using integration by parts

Definite integrals are performed by finding the indefinite integral and then using the Fundamental Theorem of Calculus. However, by default, all Mathematica warnings are switched off which can lead to problems. For example, evaluating the integral

$$\int_0^3 \frac{1}{x^2 - 3x + 2} dx$$

numerically returns the value 7.353131719997665, while evaluating the integral analytically returns the same integral. This can be overcome by setting switches to show any warnings but the default may lead to problems. Unfortunately, there is a bug in the current version, 1.6.1, when the upper limit is changed to 2 though this will be fixed for later versions.

Ordinary differential equations: Leibniz provides a clean and simple interface to Mathematica's solver using the same drag and drop mechanism. To solve an ODE such as

$$\cos(x)^2 y' + y = \tan(x)$$

select the equation and drag it on top of the expression

$$y = ?$$

Leibniz returns the answer

$$y[x] = \tan x - 1 + \frac{C_1}{e^{\tan x}}$$

Alternatively, for the equation

$$x^2 y'' + x y' + x^2 y = 0$$

select y and press shift-enter, Mathematica will return a solution in terms of Bessel functions

$$y[x] = C_1 J_0[x] + C_2 Y_0[x]$$

You can confirm that this solution is correct by dragging it back on top of original equation and simplifying. Leibniz can solve ordinary differential equations using Frobenius's method or Laplace Transforms and also handles boundary value problems.

Plotting: Leibniz makes use of Mathematica's extensive plotting capabilities, all accessible from the menu bar. Particularly impressive are its ability to do implicit function plots, the plots of direction fields for differential equations and surfaces in three dimensions. You can change the point of view for a three dimensional plot by holding down the control key, clicking on the picture in the Leibniz document and dragging to rotate it.

Exchanging Information with Other Applications

Copying and pasting text is easy, although any text you copy or paste will get transferred as straight text without formatting information. However, you can also paste in Metafiles from other applications in order to import diagrams and illustrations from drawing programs.

Exporting Information in Other File Formats: The default file format for Leibniz documents is a proprietary binary file format that is used only by the Leibniz application. The file format is platform independent, though. Leibniz documents you create on Windows will work in the Macintosh version of Leibniz. It is also possible to save Leibniz documents in LaTeX or as Mathematica notebooks. The latter is particularly useful for those going on beyond Leibniz to look at Mathematica itself.

Web Integration: In addition to the native file format, Leibniz features a range of output formats designed to make it possible to put mathematical content on the web. Leibniz exports MathML in the form of MathML 2.0 presentation tags embedded in an XML document. Leibniz also offers IBM TechExplorer and straight HTML export.

Hypertext Links: As a further convenience for creating web content from Leibniz, the application includes a hypertext link object that you can embed in Leibniz documents. When exported, a Leibniz document containing a hypertext link to one of the web formats gets saved as an ordinary link in that format. If you are creating link objects for export to one of these formats you can also type a relative URL for your link's URL.

Conclusions

Leibniz provide a very friendly mathematical word processor with extensive capabilities, **provided** you have a copy of Mathematica to hand. If you want students to get a feel of computer algebra systems then it provides a very friendly environment but it has many of the deficiencies of its big brother in a teaching context – it is excellent at doing mathematics but doesn't explain what it is doing. Perhaps the exception here is the example in Calculus (above) where students can concentrate on the concept of trying different substitutions and allow Mathematica to do the work.

My major concern is trying to establish how Leibniz might be used in a true teaching context. If it is used as a simple front end for Mathematica, in order to avoid learning Mathematica's syntax, then sooner or later the user is going to want to go beyond single line input and learn how to program in Mathematica – in which case it might be better to start straight away with Mathematica. If on the other hand, a student is never expected to go beyond the capabilities of Leibniz then it becomes a black box – throw it an integral and out pops the answer.

I am truly impressed with what Professor Gregg has achieved, Leibniz is an excellent product and very reasonably priced... if you are already a user of Mathematica.

Supplier Comments from Joe Gregg

Leibniz also includes a pair of features that make it easier to use Leibniz as a complement to the standard Mathematica front end. The first of these features makes it possible to evaluate arbitrary expressions in the Mathematica language from Leibniz. To do this, the user simply creates a paragraph in the source code format and types a Mathematica language expression. Selecting the expression and selecting Evaluate Mathematica Code from the Kernel menu sends the expression to Mathematica for evaluation. The second feature is the Notebook export command. Selecting Export/ Notebook from the File menu saves a document in Mathematica notebook format.

These two features give education users an upgrade path from Leibniz to Mathematica. Since Leibniz is very easy to learn, students can be productive with Leibniz right away. When students are ready to delve into the Mathematica language, they can start by working with simple Mathematica programs from Leibniz. As their knowledge of Mathematica increases, students can export their work to Mathematica with a minimum of effort.