Engineering teaching via MAPLE

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When he retired my PhD supervisor did some A-level marking. One day he said to me , with a sigh, "the students are not as good as in your day" and then added "but of course you were not as good as in my day"! Those of us involved in teaching mathematics have always complained about our students and I suppose that we always will, but it is fairly well established that students are less well prepared on entry now and it is more than the perception of an ageing set of lecturers [1].

One of the main aspects which is deficient is the skill in elementary algebra, but in engineering mathematics, with real world applications, algebraic manipulation is vital. These skills demand practice and training in the techniques but there is simply not time in most courses to remedy the situation. One cannot simply ignore it however since if anything, the mathematical sophistication in engineering has increased and a high level of mathematical understanding is required . For some years I have used MAPLE on second year courses to supplement the standard presentations and to provide worksheets that the students can use to look at problems that would require excessive algebra and analysis by hand. This has only been a partial success. The more able and keen students have used these worksheets enthusiastically and well, but since the material was not assessed, I suspect many students did not really use them. This is another problem. I remember learning mathematics by doing it, and formative assignments were an intrinsic part of this process. It is very difficult to get students to do any assignment that is not part of the assessment and taking into account the marking that might be involved if a lot of formative work is set, there is a disincentive for any lecturer to ask for non-summative work to be handed in.

Recently we have decided to introduce MAPLE more widely in our courses and to assess it. One advantage we have in this plan is that present day students may be deficient in algebra compared to past generations, but they are much more computer literate. Furthermore, formative assignments related to software may well be more attractive. In this article I want to share some of our plans and some of the earlier work that I have done on worksheets.

Examples from a particular worksheet

The first worksheet that I tried was on Fourier series and this was perhaps the obvious one to choose, since the effect of increasing the number of terms N can be shown on a plot examining how well the series fits the function and the Gibbs effect is easily seen. However I will describe here a less common topic: the application of Fourier transforms to linear systems and filters. MAPLE instructions introduce the convolution result that we need to connect the input x(t) and the output y(t) of a general linear system. The function k(t) is the inverse Fourier transform of the transfer function of the linear system which we denote by $\phi(\omega)$. The instruction is given with introductory text on a worksheet.

The point is now made that given any input and the corresponding output then k(t) (or $\phi(\omega)$) may be found then the system is determined for all inputs. In particular an impulse input $x(t) = \delta(t)$ leads to a very simple determination of k(t) from y(t). This is illustrated from the following instruction:

```
y(t):= simplify(subs (x(z) =
Dirac(z),int(x(z)*k(t-z),z=-infinity..infinity)));
```

$$y(t) := k(t)$$

We can then define a linear system by defining k(t) (for example $k(t) = e^{-2t}$) and examine the output for various inputs. In the instruction below the input is the Heaviside function u(t),

```
k:=t-> exp(-2*t)*Heaviside(t);
```

$$k := t \to \mathbf{e}^{(-2\,t)}\,\mathrm{Heaviside}(t)$$

```
y(t):= simplify(int(Heaviside(z)*k(t-z),z=-infinity..infinity));
```

$$y(t) := -\frac{1}{2}\mathbf{e}^{(-2\,t)}\,\mathrm{Heaviside}(t) + \frac{1}{2}\mathrm{Heaviside}(t)$$

Of course the worksheet is not usually sufficient by itself to introduce this topic and conventional lectures are required, but with sufficient computer and projection facilities the worksheet can be used within the lecture. Furthermore the students can experiment in their own time with different systems and different inputs. For use in a lecture the plot facility is extremely useful and on the same worksheet the idea of a simple filter is presented. In a set of instructions and text, an input $x(t) = e^{-t^2}$ is considered with

(a) a perfect filter where $\phi(\omega) = \delta(\omega-2)$ and
(b) an approximate filter where $\phi(\omega) = F(\omega-2)$ with

$F(\omega) = \dfrac{\sin(\omega T)}{\pi\omega}$ for some large value of T.

To illustrate the fact that for (a) all but one frequency has been filtered out, the real part of the output, c(t), is now plotted:

```
with(plots):
plot(c(t),t=0..4*Pi);
```

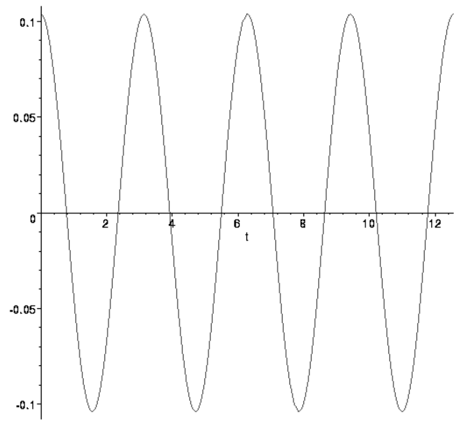

Fig 1: Maple output for a perfect filter

In the case of the approximate filter we must first illustrate the form of the function, show that the area under the curve is unity and that it peaks for large T. To do this we set T=10 and perform the following set of instructions: (i) define $\phi(\omega)$; calculate the integral of $\phi(\omega)$ from $-\infty$ to ∞; plot $\phi(\omega)$ over a significant interval.

Then, taking $\phi(\omega) = F(\omega-2)$, the corresponding plot of c(t), the real part of the output is:

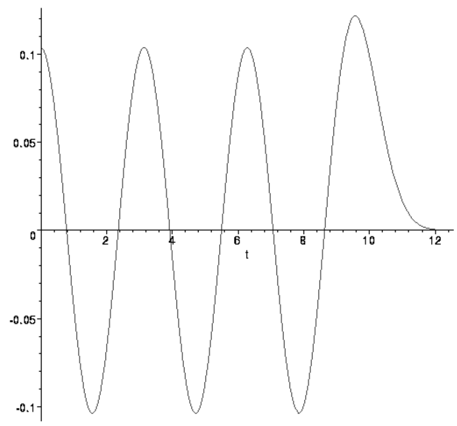

Fig2: Maple output for an approximate filter

which shows that it is an approximate filter with the amplitude and phase varying only for large t.

The initial experience

One of the problems with using these worksheets is that the student needs to become familiar with the syntax. Whilst this is fairly straightforward for the modern, computer oriented student *if enough time is spent on them,* the fact that the worksheets have constituted formative rather than summative work has led to many students not developing the skills. We hope that this will not be the case in the future since we are now introducing MAPLE in the first year. All the MAPLE worksheets will be assessed and they are planned to consist of :

```
expand(R*cos(x+a));
```

$$R\cos(x)\cos(a) - R\sin(x)\sin(a)$$

```
x:=evalf(solve({cos(x)-R*cos(x)*cos(a),sin(x)-R*sin(x)*sin(a),R>0}
,{R,a}));
```

$$x := \{a = .7853981634 + 6.283185308\ \_Z1\sim, R = 1.414213562\}$$

```
x[1];
```

$$a = .7853981634 + 6.283185308\ \_Z1\sim$$

```
op(2,x[1]);
```

$$.7853981634 + 6.283185308\ \_Z1\sim$$

```
op(1,op(2,x[1]));
```

$$.7853981634$$

Fig3: Maple trigonometry example

(a) an introduction to MAPLE introducing most commonly used syntax and revising basic algebra, trigonometry and matrices;
(b) a worksheet on vectors;
(c) a worksheet on statistics.

In worksheet (a) the students will be expected to hand in the completed worksheet with additional questions attempted and explanatory text given. For example the set of instructions shown in Fig 3 is a routine for expressing cos(x)-sin(x) in the form Rcos(x+a) and the syntax of the sequence function and op function are used to choose a particular part of the solution.

It is expected that the trigonometry theory has been covered prior to this worksheet and that the object is both to enhance the understanding of this theory and to introduce the syntax for using the sequence notation and the op function. One of the questions that the student needs to do to complete the worksheet will then be:

Consider the formula p = cos(2x) - 3sin(x). Express this in the modulus and phase form Rsin(2x+b) finding R>0 and b. Use the sequence and op instruction to pick out one value of b. Insert text to explain in your own words the actions of the sequence and op functions that you have used.

Worksheets (b) and (c) are designed in the same way except that the student is not expected to have covered the theory before attempting the worksheet. Consequently much more information is given and these worksheets are to be discussed in the lectures and tutorials.

The final experience

The skill of teaching oneself some mathematics using a package such as MAPLE is valuable. In the career of an engineer many different mathematical theories will be important at different times and for different projects. It is quite likely that a topic has not been covered in the limited time available to study mathematics - or of course it may have been forgotten! Even in final year projects it is quite usual for some new mathematical theory to be important. Quite recently there has been interest from students at Staffordshire University on Bessel functions, and these have not been covered in the main mathematics syllabus for many years. Using the help facilities in MAPLE it was possible to glean all that was required which was the shape of the curves and the roots. Provided the student is familiar with the plot function and simple programs a large variety of mathematical results are available. Thus it is important to get the basic skills and understanding secure and then MAPLE (or a similar package) can be used to extend the available mathematical tools. Perhaps, even a reactionary such as I can conclude that this is progress, in that we are teaching the students to understand the basic ideas rather than a box of tricks. Although I still think the box of tricks is fascinating and challenging. Of course if the material in MAPLE is not sufficient for the student to understand all that is required about a topic like Bessel functions then there are websites which give further information and they can even, (dare I say this?) read a book.

Reference:

[1] Engineering Council (2000), *Measuring the mathematics problem*, Engineering Council, London