How Should a Perfect Computer Aided Assessment Package in Mathematics Behave?

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TREEFROG: A Natural Algebraic Interface?...With the TREEFROG project we have long been seeking to provide a mathematical assessment tool which allows students to type in mathematics in a way which is a similar as possible to pencil and paper, but with a certain "intelligence" built in. I am sure we have not succeeded fully, as I shall explain below; but I would like to initiate (or, more correctly, continue) debate in this area with a view to agreeing as a community what is desirable in such an environment. Please feel free to give your responses to the questions raised here, as described at the end of the article.

As I see it there are two main areas of concern; the input and display of mathematical expressions, and the faithfulness of systems in recognising a "correct" response in the same way that a lecturer or teacher would.

Input and Display

With the growing use of IT throughout the curriculum, almost all students of mathematics in Higher Education will have used a spreadsheet at some point. Almost invariably these require the linear input of expressions, as in
(-b1 + sqrt (b1^2 + 4*a1*c1))/(2*a1)

With practice one can become adept at entering such expressions, but even experts may have to think twice if they mistakenly enter

.../2*a1 instead of .../(2*a1).

My own feeling is that students who are less than secure in the manipulation of fractions must have access to a system that displays them in the conventional two dimensional format, as the extra mental effort of translating to linear form must distract them from the mathematics involved. To date, TREEFROG still does not have this capability.

Having said that, a lot can be done with linear input if fractions are not required. The normal text font in DOS and Windows has always had a few mathematical symbols such as powers up to 3, and '±'. With recent versions of Windows, this font also includes an integral sign, Greek letters, and several more besides; try inserting a symbol from this font in Word if you would like to see more. Software such as DERIVE and AUTOGRAPH define their own character set, which is probably the best option for mathematical applications as students do not have to wade through all the characters for different languages before they reach the symbol they need.

If a two dimensional display is used, then there are different methods that can be used to input an expression. A simple but effective method is used in Heriot-Watt's "Past Papers" (and CALM before that), whereby the student enters a linear expression which is simultaneously displayed in two-dimensional format. In using this tool a student can learn to use the ubiquitous linear input form, while avoiding some of its pitfalls.

Another common approach is used by DIAGNOSYS and the Equation Editor within WORD, whereby clicking a button opens up a template for a mathematical object such as a fraction. TREEFROG also uses templates, but slates these directly into linear form as seen in Fig 1

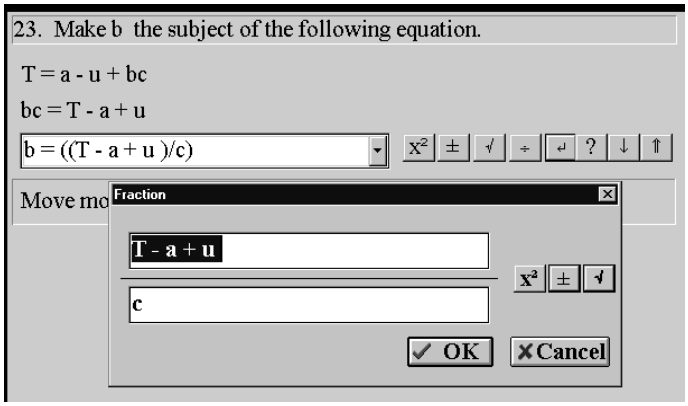

Fig 1 TREEFROG translates templates to linear form

Using a button to enter a common symbol, such as 'x', seems to me to slow the user down and take up valuable screen space, thus detracting from the learning experience. Those who are inexperienced in typing might find such an approach more helpful.

Given that we can expect our students to be more and more familiar with keyboard entry, the ability of a package to input a complex expression without using the mouse is important for speed and concentration on the problem in hand. But what should happen when the student enters a character such as '/'? If the intention of the software is to display mathematics exactly as prescribed by the user, as in TeX or Microsoft Equation Editor, then no special action need occur. If we are aiming to support a novice student of mathematics, then it seems reasonable to expect that a two-dimensional display will appear automatically. So if the student types "1+2/", we may expect this sort of display (as seen in DIAGNOSYS, for example)

$$1+\frac{2}{}$$

with some form of cursor appearing under the '2' to invite input of the denominator. Normally any expression typed now will form part of the denominator, until a special key (cursor, for example) is pressed. So entering the sequence of characters "1+2/3+4" using such an input tool will result in the display

$$1+\frac{2}{3+4} \text{ rather than } 1+\frac{2}{3}+4$$

which might confuse students who also used linear forms of expressions. A similar example I have come across is where the keyboard input "3x2" is translated to $3x^2$; again, one could ask whether this conversion is desirable for novice mathematicians.

Is the answer right?

It has long been recognised that it is a complicated business recognising the correct answer to an algebraic question. String comparison ignores the commutativity of operators, and redundancies such as extra parentheses. Evaluating expressions at various values of the unknowns is much more successful, but ignores the structure of an expression almost entirely. It is entirely possible, for example, that if a student is asked to divide $x^2 - 3x + 2$ by $x-2$, the answer

$$\frac{x^2-3x+2}{x-2}$$

will be accepted as correct. Only if the value $x = 2$ is included in the evaluation routine will this result be rejected. Even it is not accepted, something is lost, as the student's correct understanding of what the question was asking her to do has seemingly been contradicted by the computer.

TREEFROG attempts to distinguish between acceptable interim steps in a question and allowable final answers. Interim steps are checked, by applying mathematical rules to a known correct expression and the student's input. If a common "normal form" is reached (such as expanded polynomials with like terms gathered), then the step has been proved correct. This should ensure that any correctly applied mathematical knowledge is not contradicted. The final step is recognised using matching, which takes account of associative and commutative operators and ignores redundant syntax, ensuring that the student has completed the required task. Thus, if the question is to solve the linear equation $2x + 5 = 8$, the steps

$2x = 8 - 5$, $2x = 3$, $12x + 32 = 50$, $x = 1.5$, $x = 3/2$

will all be accepted as mathematically correct; but only the last two will be recognised as a final answer to the question. When the equation to be solved is quadratic, there may be even finer distinctions to be made. For example, $(x-1)^2 = 0$ has only one solution, so can be seen as equivalent to the equation $x = 1$. As an auxiliary equation for a second-order linear differential equation, however, we need to keep hold of the fact that there is a repeated root. Rational functions also present complications, as we may or may not wish to exclude the possibility where the denominator is zero; when re-writing as partial fractions we would not worry about this, for example.

Discussion – Over to you!

We will use the discussion list maths-caa@jiscmail.ac.uk to enable discussion on the points raised here. You can join the list by following the third link on the page www.jiscmail.ac.uk/lists/MATHS-CAA.html. Here are some suggested starting points.

1. A user interface should be

- Natural, allowing you to work in a similar way to your normal paper-and-pencil approach.
- Flexible, so that it caters for different levels of mathematical and computing experience.
- Supportive, providing help when required.
- Consistent, so that the same task is executed in the same way in all contexts

For the purpose of helping students to learn mathematics, which interfaces have supported them best? Which interface issues have caused most difficulty in your classes? Would it be possible as a community to describe how an ideal mathematical input tool should behave, especially with regard to keyboard and mouse input?

2. In deciding whether a mathematical expression entered into an assessment package is a correct answer, how much control do we need as lecturers? Is multiple choice enough? Or do we want software to be able to exercise discretion in the same way we would in marking work by hand? If so, how would you envisage that such discretion might be controlled?

If you are interested in following up on any software mentioned in this article, I suggest you visit the LTSN Maths courseware database at
http://mat-nt2.bham.ac.uk/delta/.

Letters to the Editor...

Received From:
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MathSciNet, the mathematical database published by the American Mathematical Society, provides web access to the signed reviews and bibliographic data from *Mathematical Reviews* and *Current Mathematical Publications*. Its coverage goes back to 1940, the year when *Mathematical Reviews* commenced publication.

In order to encourage its increased use by the mathematical community, AMS offers consortia pricing for *MathSciNet*. There are now 95 consortia worldwide that enjoy access to *MathSciNet* at preferential rates, including a consortium that has recently been set up in the United Kingdom. At present membership comprises about 15 universities but there are hopes that this will increase. It is administered from the University of Wales Swansea.

The subscription to *MathSciNet* is made up of two elements. There is the Database Fee (DAF), or "content fee", which reflects the cost of creating and maintaining the *Mathematical Reviews* database — over 47,000 items added annually. In 2002 this will be $6,141, the same as in 2001. In addition there is the *MathSciNet* fee itself, which is based the concept of 'Mathematical Activity' (MA). MA is the number of papers reviewed in *Mathematical Reviews* in a three-year period which originate from that site.

The consortium DAF is based on the number of previous subscribers in the group and is calculated at the same rate as for individual sites; thus if there are ten members in a consortium, and they were all previous subscribers, the consortium DAF would be $61,410. If completely new members join, there is no increase in the total consortium fee.

The formula for the *MathSciNet* fee is MA x $20 + $250, with a minimum fee of $250 and, for consortium members, a maximum fee of $1000. For single subscriber sites the MSN fee will be a flat $1,865 in 2002. Thus a site that already subscribes to *MathSciNet* stands to gain about £600 by joining a consortium.

The philosophy behind the programme is, of course. to encourage non-subscribers to join, and to encourage subscribing sites to persuade them to join, since in that way their share of the consortium DAF will be reduced. With this in mind it has been agreed that completely new subscribers to *MathSciNet* who join the UK consortium will pay 10% of a site DAF in the first year ($614) and 50% in the second year before they are asked to pay their full share in year 3. They will of course each have to pay their own share of the MSN fee. It is hoped that this will ease the transition to the full rates for such sites.

The scheme is described in detail on the AMS website http://www.ams.org/