Curriculum 2000
Report of a briefing day

Few people will be unaware that major changes have taken place in the A level system. Although the content of the mathematics A level itself has changed little, as the following articles report, there are a number of changes of which staff at universities should be aware. Similar issues are also arising in Scotland following the "Higher still" review.

These articles, which originally appeared in *MSOR Connections* 1(3) and 1(4), report on these changes. Three follow from talks given at a briefing day which took place on Monday 11 June 2001 at the Institute of Education, University of London. The purpose of this was to explain the changes to the A level system under *Curriculum 2000*. One details the situation in Scotland.

I would like to thank all those who attended the meeting, and particularly the speakers whose excellent talks generated interesting questions and stimulating debate. I would particularly like to thank Jack Abramsky, the principal subject officer for mathematics at the QCA, who gave two talks and without whom the day would have been much diminished.

Chris Sangwin, LTSN Maths, Stats & OR Network

Mathematics: the national scene

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We begin with a mathematical overview, starting at primary school. The most significant recent change here is the National Numeracy Strategy. This was set up as part of the government's "raising standards" agenda, with the primary strategy implementation which began in schools in 1999, including a Framework for teaching mathematics from reception to year 6. This is a year by year teaching programme to deliver the key stage 1 and key stage 2 programmes of study of the mathematics National Curriculum. The Framework is a suggested implementation of the National Curriculum.

The National Strategy for key stage 3 goes national in September 2001. The key stage 3 Framework for teaching mathematics for years 7, 8 and 9 is a year by year teaching programme to deliver the key stage 3 programme of study of the

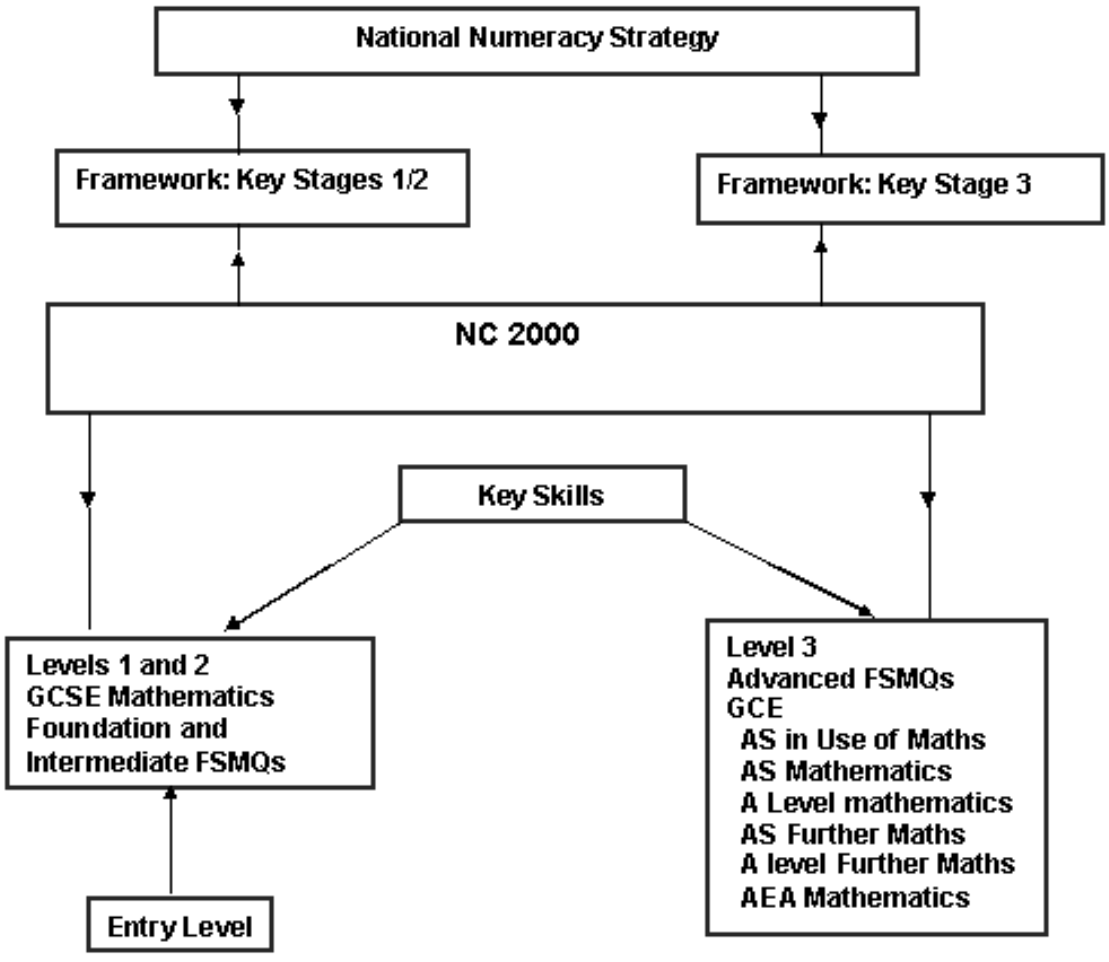

Figure 1: A mathematical overview

mathematics curriculum. This is available from the DFES, and is published by Cambridge University Press. Key objectives for each year are the basis of the framework, which also contains numerous examples and suggestions for progression in mathematics. As for KS1/KS2, the KS3 framework is a suggested implementation of the National Curriculum for mathematics.

Schools began teaching from the revised National Curriculum (NC2000) in September 2000. NC2000 includes differentiated programmes of study for KS4 which is examined externally through GCSE. These papers are set by the three English awarding bodies (AQA, Edexcel and OCR). GCSE's are set also by WJEC for Wales and CCEA for Northern Ireland. (Note however, that the National Curriculum for Wales now differs from the National Curriculum for England. Northern Ireland has its own Curriculum.)

The first GCSE's based on NC2000 will be awarded in 2003, with the first teaching yet to commence in September 2001. In the new national qualifications framework GCSE straddles level 1 and 2 qualifications.

Mathematics beyond GCSE, but pre-university level, is classified as level 3. Table 1 lists the level 3 qualifications that will be available from 2002.

Level 3 mathematics qualifications available under NC2000

GCE:
AS mathematics
A level mathematics
AS further mathematics
A level further mathematics
AS Use of mathematics (*this is new - see page 28*)
Advanced extension awards in Mathematics (AEA)

also:
Advanced Free Standing Mathematics Units (FSMUs)

Table 1: Level 3 mathematics under NC 2000

Key skills, which includes "application of number", exist at levels 1,2 or 3. Entry level qualifications are pre-level 1 qualifications (for post-16 year olds). Key skills at level 3 attract UCAS points. Problems arise in seeking to establish equivalencies between the basic and key skills qualifications and GCSE/GCE qualifications.

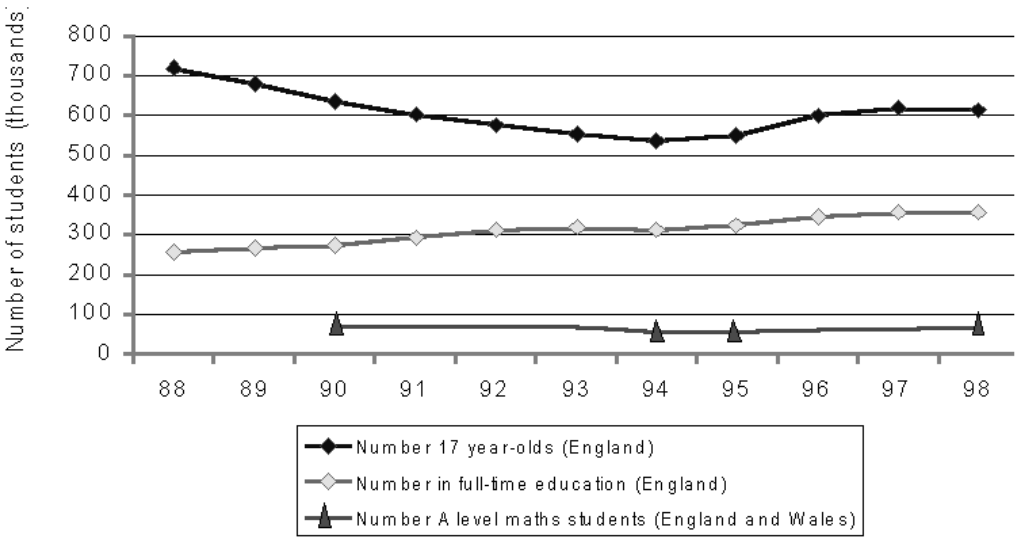

Figure 2: UK post-16 participation in full-time education

UK post-16 participation in full-time education

The graph in Figure 2 shows trends over the last decade or so. The percentage of the cohort studying mathematics beyond GCSE has declined for those in full-time education. Numbers of A level mathematics entries have remained fairly static over the last ten years at about 10% of the entire age cohort. Qualifications other than A level mathematics are needed beyond level 3 to boost the number of students studying mathematics beyond GCSE. This has resulted in the development of free standing mathematics qualifications (FSMQs) and a new GCE AS in *use of mathematics*. FSMQs have also been developed at levels 1 and 2.

GCE A level mathematics

The new syllabuses from September 2000 require that GCE A level mathematics is made up of 6 units of assessment including AS and A2 modules. Modules are designated as AS or A2 in the new world in which the government wishes students to broaden the range of their studies by taking four AS subjects in year 12, progressing three of these into full A levels in year 13, and possibly doing an additional AS in year 13. Since AS mathematics was designed as the first half of the course in A level mathematics, the anticipated mode of delivery was for completion of the AS in the fourth term of study for the majority of candidates.

There are seven approved specifications for A level mathematics and AS mathematics. These specifications can be obtained directly from each of the awarding bodies. Information about the English specifications may be obtained from the web sites of the awarding bodies: www.aqa.org.uk, www.edexcel.org.uk and www.ocr.org.uk

All A level mathematics have the pure mathematics core which accounts for 50% of the assessment; at least 25% of the assessment is application (in mechanics, statistics or in discrete mathematics) but no single application need be studied. No more than 20% of the total marks are for coursework and a synoptic element worth 20%. At least 25% of examinations are with the use of weak scientific calculator only. The AS core pure mathematics is half the A level core. The assessment should be a minimum of 8-9 hours examination time for the AS+A2 and 4-4.5 hours of time for the AS.

Elements of the 1983 core have been reintroduced and there is more emphasis on correct use of notation, algebraic manipulation, logical deduction and proof. There are to be more multi-step problems and fewer structured questions. The prerequisite knowledge is to cover expectations for the upper grades at GCSE (B-A*). The syllabus specifies a list of formulae candidates will be expected to know and a common list of allowed formula. The syllabus also gives grade descriptions for grades A, C and E together with approved minimum mark weightings for five assessment objectives: knowledge, understanding and skills (KUS) (30%); constructing rigorous mathematical arguments (30%); solve realistic problems (10%); use, read, present and interpret maths (5%); use ICT (5%).

Significantly students will now only be allowed to re-sit each unit once. There are related AS qualifications in applied mathematics, mechanics and other module combinations.

Free standing mathematics units

Foundation units: level 1
- Managing money
- Working in two and three dimensions
- Making sense of data

Intermediate units: level 2
- Calculating finances
- Solving problems in shape and space
- Handling & interpreting data
- Making connections in mathematics
- Using algebra, functions and graphs

Advanced units: level 3
- Working with algebraic & graphical techniques
- Using and applying statistics
- Modelling with calculus

Table 2: Free standing mathematics qualifications

Each of the titles in Table 2 is a free standing mathematics qualification (FSMQ) in its own right. Each is a sharply focussed aspect of mathematics catering for individual students' needs and designed to support other areas of study through developing relevant mathematical skills, understanding and knowledge. Students produce portfolios of work and sit externally assessed written examinations, with equal weighting between the two components. Applications of mathematics are stressed, as is the use of ICT to process real data. Typically, an FSMQ requires about 60 hours of teaching time.

Advanced extension awards

This award is designed to replace the current 'S' papers in mathematics. The development of this award was specifically influenced by the report "teaching and learning algebra pre-19" published by the Royal Society and the Joint Mathematics Council of the United Kingdom.

The government's remit is that:...*"The advanced extension award should:*

- *stretch the most able advanced level students by providing opportunities for students to demonstrate greater depth of understanding than required in A levels;*
- *ensure that the most able students in this country are tested against standards compatible with the most demanding to be found in other countries;*
- *be accessible to all able students, whatever their school or college and whichever specification they are studying, so that significantly more young people have the opportunity to take them than take existing special papers;*
- *help differentiate between the most able candidates, particularly in subjects with a high proportion of A grades at A level, in order to obviate the need for universities to develop their own entry tests;*
- *complement the other world-class tests being developed for the most able nine and thirteen year olds."*

Specifically the aims of the advanced extension award in mathematics are to provide a sense of achievement and a stimulating mathematical challenge through:

- Encouraging students to use what they have been taught;
- encouraging students to think beyond what they have been taught;
- encouraging students to develop confidence, stamina and fluency in working through unstructured problems which might demand multi-step analysis or the exploration of different possibilities;
- building chains of logical reasoning and using concepts of proof;
- testing critical thinking;
- stretch the most able students of mathematics by requiring a greater depth of understanding than is required in GCE A level mathematics;

- be designed to help universities discriminate in selecting able students according to their mathematical ability;
- be accessible to all students of grade A standard in GCE A level mathematics, but be of an appropriate level so that the pass grades of merit and distinction are achieved by about 1/3 of this cohort;
- help develop specific skills beyond those required for A level, in preparing students for university courses in mathematics and in subjects which rely heavily on mathematical analysis.

Final note

The new developments have been put in place to address concerns that have surfaced in the mathematics community and other end users of mathematics. Change takes time to work its way through the system. A child who started school in 1999, when the National Numeracy Strategy was first introduced, will not go to university until at least 2012, and may not graduate until 2016. Such a child should reap the benefits resulting from all the changes introduced into the system right now. It takes many years for change to be fully felt, and by then there will be still further changes introduced into the system. Some of the changes may have a more immediate effect.

Mathematics in Curriculum 2000: What will students know? What will students not know?

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September 2000 saw the introduction of a package of changes to the post-16 curriculum in England and Wales under the banner of *Curriculum 2000*.

- All AS and A Levels became modular with a credit rating of 3 modules (or units) for AS and 6 for A Level.
- Students are expected to complete at least 4 AS Levels in Year 1 as part of broadening of their curriculum. In Year 2 they are expected to continue with 3 subjects to full A Level standard.
- Full funding in FE now depends on the delivery of Key Skills. Schools are also expected to deliver them.
- There is a new style of GNVQs, making it easier for students to study a mixture of academic and vocational courses.

This paper looks at the situation of mathematics following the first three of these changes.

AS and A Level course structure

There are now 5 AS/A Level Mathematics syllabuses (now called specifications) in England, and in addition those from Wales and Northern Ireland. While there are considerable differences in philosophy between the syllabuses, they nearly all have essentially the same structure, with strands of modules in Pure Mathematics, Mechanics, Statistics and Discrete Mathematics.

No two syllabuses have exactly the same set of modules but in general the differences between them are on the periphery and more likely to affect Further Mathematics candidates than those doing the single A Level. The scheme diagram for our MEI scheme is shown in Figure 1; we have a few more modules than some of the other syllabuses, including a strand in Numerical Analysis.

The first level modules, those numbered 1, are classified as AS modules, the rest as A2. In most other subjects the AS Level consists of 3 AS modules; you then take 3 A2 modules to complete the A Level. In mathematics it is not as simple as that and it is simplest to forget all about the distinction, just thinking of them all as modules.

At first sight it looks as though there is an enormous number of possible combinations for AS and A Level but this is not the case.

A Level

For A Level Mathematics all students must cover the common core (now called the subject criteria). This defines the overall content of the modules *Pure Mathematics 1*, *2* and *3*. It is the same for all syllabuses. A summary of what is in and what is beyond the single A Level is given in Appendix A on page 15.

Whereas there is no variability in the pure half of the A Level, the same is definitely not the case for the remaining 3 modules. Most students do 3 applied modules and these can be drawn from one strand (e.g. *Statistics 1*, *2* and *3*) or two (e.g. *Mechanics 1* and *2*, *Statistics 1*). However it is also possible for a student to do 1 extra pure module (typically *Pure Mathematics 4)* and only 2 applied modules.

Why is there such variability in what students do for the applied part of the single A Mathematics Level ? The answer is that mathematics is a service subject for many others. Quite different applied mathematics is needed to prepare students for university courses in, say, engineering, biology and management. It would however be very helpful for schools if university departments gave clear advice as to what applied modules they would like to see future undergraduates taking in preparation for different types of courses.

AS Level

There are a number of possible AS Level certifications. The two most likely to be taken are Mathematics and Applied Mathematics. Statistics may also prove quite a popular AS Level.

- AS Mathematics students will do *Pure Mathematics 1* and *2* and one of the first level applied modules, *Mechanics 1*, *Statistics 1* or *Decision & Discrete Mathematics 1.*
- AS Applied Mathematics consists of *Pure Mathematics 1* and two out of the three first level applied modules.
- There are several routes to AS Statistics, all involving *Statistics 1* and *2.*

Students taking AS Mathematics will thus have done more pure mathematics than applied and this will include the AS part of the common core. This too is common to all syllabuses. There will however probably be rather greater variability in the content covered by those doing AS Applied Mathematics.

Concern about the content of AS courses may however be misplaced. There is funding and timetabling pressure on schools to complete an AS of some description in one year. Many of those who record an AS qualification, including Applied Mathematics and Statistics, on their UCAS forms will go on to complete A Level Mathematics.

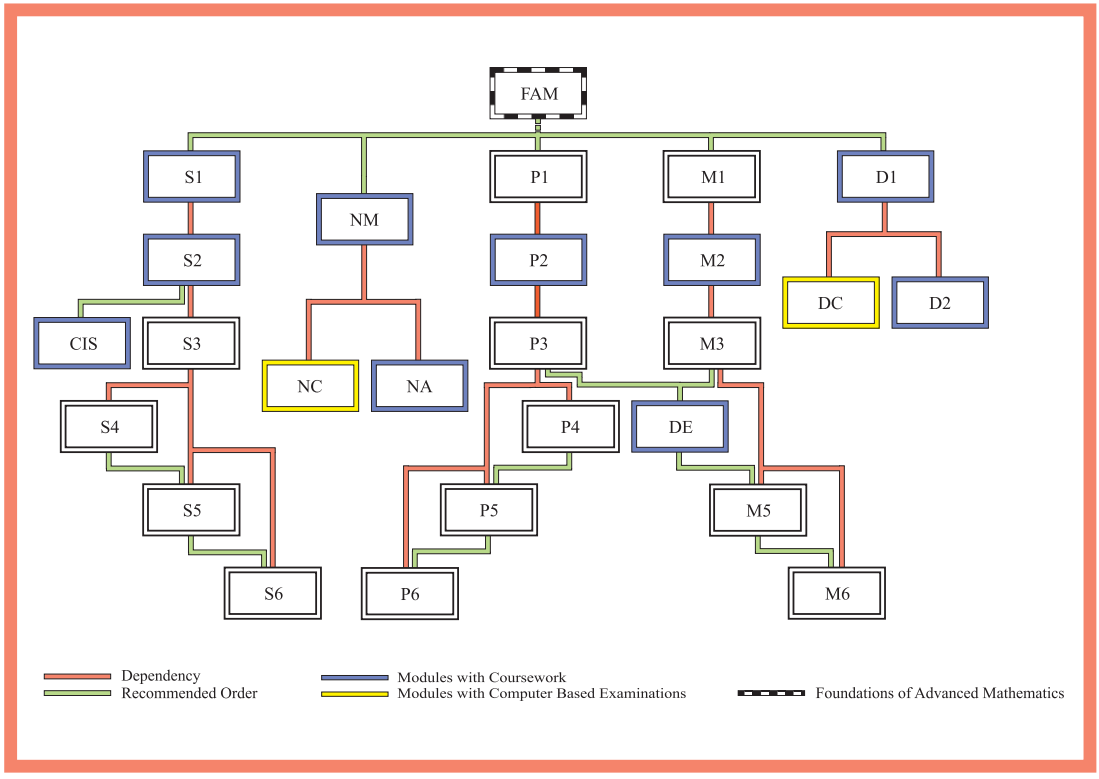

Figure 1: Module dependency for MEI mathematics

Further Mathematics

The regulations have been tightened concerning Further Mathematics so that those doing it at AS level must now include *Pure Mathematics 4*, as well as *Pure Mathematics 1-3*, in their 9 modules. For the full A Level Further Mathematics (12 modules), *Pure Mathematics 5* is also required. Although there is no common core for Further Mathematics there are topics, such as complex numbers, which can in practice be guaranteed.

Sweat-shop sixth forms

Looking at the syllabus and examination structure would thus lead you to expect that students will arrive at university with well-defined knowledge of pure mathematics. Will it really be like that ?

The first point to make is that there has been very little change in mathematics. Nearly all A Level students were already following modular syllabuses. The content has hardly changed at all; it was already tightly proscribed by a core that was very little different from the new one. So there is no reason to expect any sudden improvement in the quality of students arriving at university.

What has changed is the curriculum in which this syllabus is now embedded. Until now most students doing mathematics were also taking two other A level subjects through their two years of sixth form. That is no longer the case. Their workload in the first year has been increased by the introduction of one or two extra subjects, and they are also expected to take Key Skills. Students now have much more work to do with less time to do it in and reduced teaching time per subject.

The situation in mathematics is even worse than that. Over the 10 years that modular syllabuses have been running, it has been found that the most satisfactory pattern of A Level entry is to do 2 modules in the first year and 4 in the second. In effect there was a vast experiment, involving thousands of schools and colleges and hundreds of thousands of students, over a ten year period; 2 + 4 was the outcome. This gives more time to lay solid foundations. However in many schools and colleges mathematics departments are now being expected to deliver 3 modules instead of 2 in the first year.

In most subjects the AS modules have been made reasonably easy and short. In mathematics we were not allowed to do this. Uniquely to mathematics, there was already an AS core in place and this was toughened rather than relaxed for *Curriculum 2000*.

Reports from schools and colleges are frightening. It seems we are moving far away from education into sweat-shop sixth forms. The quotes highlighted on this page are typical of what many teachers are now saying.

My own view is the problem is not with the mathematics syllabus, but with it being used in conditions for which it was not designed. The decision to expect students to do more than 3 subjects in the first year was a political one, taken after the essential features of the syllabus, like the common core, had been irrevocably decided.

It is a case of trying to get something for nothing. On the one hand the standard of A Level students must be the same as before and students must still leave school with 3 of them. On the other hand they must do extra AS Levels and Key Skills as well.

I am all for a broader curriculum but it can only be successful if it is done honestly.

"I can't teach properly any more. For example, I don't have the time to explain why the derivative of e^x is e^x. I know I should but instead I just have to tell them to accept it."

"Time pressure is forcing me to move on to the next topic even though I know that some of the students have not understood this one."

"There is no time for any enrichment material. All we can do is focus on the sort of questions that will come up in the exam."

"As a result of the pressure, student depression is now a serious issue in our college."

"We expect 50% of our AS students to drop maths at the end of the first year."

"Maths is now much harder than other subjects."

"We can't afford the time to allow students out to go to university open days."

"We have had to cut extra curricular activities, like the Duke of Edinburgh's award. There's just not time any more."

Implications for universities

What I have just said will have very serious implications for universities, and particularly for numerate degree courses. It is entirely predictable that the mathematics of those who have taken A Level will be weaker than at present, and it seems likely that fewer will have completed the full A Level. There are however things that universities can do, through their admissions procedures, to help themselves and schools.

AS or module grades

Many students will obtain AS grades in Mathematics, Applied Mathematics or Statistics this summer and will then be required to enter them on their UCAS forms. However these grades cannot be relied on to provide accurate information about the students or their potential. There will be good students with only moderate grades, people who just have not had the time to do the AS course properly and show what they can do. Many of these will re-sit the modules in January and then get grades which provide a much more accurate description of their ability; others, sadly, will give up mathematics altogether.

GCSE grades may well prove more useful to admissions tutors than those from one-year AS Levels. However a better alternative would be for universities to look at module grades in mathematics, rather than AS grades, and to let it be known to school that this is their policy. This will feed back into school practice.

Of all the problems with *Curriculum 2000*, the one that affects mathematics most adversely is the expectation that students will do 3 modules in the first year. This is not a legal requirement and it is still possible for schools to follow the 2 + 4 model.

It is entirely realistic to think of a student who would get 2 grade A modules (*Pure Mathematics 1* and *Mechanics 1*) under the 2 + 4 regime, but under 3 + 3 would come out with 3 grade C modules (*Pure Mathematics 1*, *Mechanics 1* and *Statistics 1*) and a grade C AS Level in Applied Mathematics.

Compare the information that these two scenarios provide for admissions tutors. The 2 As show that the student may well have the potential to do very well at university. The grade C AS gives a very different picture, that of someone who is unlikely to sparkle. It is clearly to universities' advantage to get the most reliable information they can. This can be achieved by asking for a small number of module grades rather than that for an AS. If universities adopt this policy, many schools will then revert to the 2 + 4 policy for mathematics, knowing this to be in their students' better interests.

The new UCAS tariff

The pressure on schools to complete an AS level in one year can be seen as symptomatic of a way of thinking that mathematics is only of value if studied in blocks of 3 modules.

Another, and very damaging, example is the refusal to allow credit on the new UCAS tariff for single mathematics modules (with the exception of QCA's free-standing units).

Further Mathematics

Another negative effect of *Curriculum 2000* is that it has caused a decrease in the numbers of students taking Further Mathematics (as a fourth subject). This will seriously disadvantage undergraduates in certain degree courses, particularly mathematics, physics and engineering. It would really help schools if, in such cases, prospectuses included a form of words such as:

"Although Further Mathematics is not a requirement for entry to this course, those who have taken it, at AS or A Level, are likely to find the course more accessible."

A further caution: dual certification

QCA and the examination boards have recently come up with a set of rules regarding the number of certificates a candidate may be awarded. This will mean that the same modules can contribute to two apparently independent certificates. The regulation reads as follows:

> **Number of awards:** The maximum number of certificates for which a candidate is eligible is as follows:
>
> | | |
> |---|---|
> | 3 units | one AS certificate |
> | 6 units | two certificates (one AS + one A **OR** two AS) |
> | 9 units | three certificates (two AS and one A **OR** three AS) |
> | 12 units | four certificates (two AS and 2 A **OR** three AS and one A **OR** four AS) |
>
> Exactly 3, 6, 9, 12 units must be used when claiming 1, 2, 3, 4 certificates, respectively.

I cannot see how this apparent double counting will fail to cause confusion to end-users, such as employers and admissions tutors.

Key Skills

Many *Curriculum 2000* students will be presenting Key Skills certificates to admissions tutors. Although there are six Key Skills, only three of them (Communication, Application of Number and IT) accrue UCAS points and it will probably be unusual to see applications that include any of the other three (Problem Solving, Working with Others, Improving own Learning and Performance).

Key Skills are available at different levels. Level 2 is intended to support work in other subjects at GCSE, Level 3 at A Level or Advanced GNVQ and Level 4 university work. Thus Application of Number Level 2 might be the level of mathematics required for GCSE geography, and so on up.

One of the curiosities of the new UCAS tariff is that it gives points for Level 2 Key Skills. It is hard to see the rationale for this. Those applying for university have been doing A Levels or Advanced GNVQs and so they should have been using Level 3 Key Skills. Level 2 is not designed to add value to a university application.

My own view is that of the three common Key Skills, working for Communication will be of real value for many future undergraduates, and the same will often be true for IT. Application of Number, however, is likely to present problems for all concerned, students, teachers and end-users.

Those who do AS or A Level Mathematics should be presenting Level 4 Application of Number (their mathematics should support university level work in non-numerate subjects). Other A Level students should, as already explained, be presenting Level 3. However most university applicants will be one level down on that.

There are two parts to the assessment, portfolio and external test. The Level 4 external test has not been available this year but even if it had, it is very unlikely that many students would have taken it. The reason for this is that the Application of Number tests that have been set (by QCA), at Levels 1, 2 and 3, have all been much too hard, resulting in very low pass marks and very low pass rates. The rationale for the Key Skill, supporting work at the appropriate level, does not seem to have been taken into account in setting the papers.

It is possible to avoid the Application of Number external test by claiming exemption, and this looks likely now to become common practice. However the exemptions all work one level down from that which the students should be offering. AS or A Level in a mathematics based subject gives exemption at Level 3, GCSE (C or better) at Level 2.

There have not been similar problems with Communication and IT and so it seems likely that many of those presenting Key Skills will be at their correct level in those two but one down in Application of Number.

The actual content of Application of Number is quite trivial, a subset of Intermediate GCSE at Level 3, and even less at Level 2; the portfolio demands are somewhat greater, however.

My own view is that it is really important for students to acquire certain skills, particularly communication and team-working, and that consequently sixth form courses should be designed to foster these skills. However I remain to be convinced of the validity of assessing them.

Conclusion

Although *Curriculum 2000* has been devised with the best of intentions, it is nonetheless the case that the overall effect is to damage mathematics. We should not be prepared to accept this situation. Our whole future as an advanced economy depends on science, technology and engineering, and all of these require both a high standard of mathematics and sufficient people attaining it. Otherwise we will bequeath a third world country to our children and grandchildren.

Appendix A: Pure Mathematics - A Level Content

	Single A Level	**Further Mathematics**
Calculus		
Differentiation	Product, quotient & chain rules Trig functions Exponentials & logarithms Implicit functions	Inverse trig functions
Integration	Integration by parts Simple substitution Trapezium rule	Repeated applications Trig substitutions Simpson's rule
Differential equations	Separable variables	2nd order linear equations Integrating factor method
Trigonometry	Compound angle formulae Solution of trig equations	Factor formulae
Exponentials/logarithms	Algebraic manipulation	Hyperbolic functions
Algebra	Sequences & series General binomial expansions Partial fractions	Proof by Induction Complex numbers
Geometry	Curve sketching Parametric equations	Oblique asymptotes Polar co-ordinates Conics
Vectors	Points and lines Scalar product	Planes Vector product Matrices

AS/A Level Mathematics modules and the UCAS tariff

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In the proposed new UCAS tariff, no credit is to be given for individual AS/A Level modules. The purpose of this paper is to argue that in the case of mathematics this position is contrary to the interests of those most concerned.

- It is not uncommon for courses in Higher Education Institutions to be undermined by the inability of their undergraduates to use mathematics as a service subject.

- Many school students, including those in identifiable groups, take one or two mathematics modules but not the three needed for AS certification. The number of these students is likely to increase.

- Students who have taken such modules are better placed to meet the demands of particular Higher Education courses. Information about these modules is relevant to admissions tutors and should be part of any overall profile.

- The effect of giving credit for individual modules would be to increase the numbers of students doing some mathematics (as opposed to none) post-GCSE.

The needs of Higher Education

There are some subjects, such as engineering and physics, where the need for mathematics as a service subject has long been recognised. This paper is not only about them, but also about the many other disciplines where it is now impossible to design undergraduate courses which meet the necessary academic standards but make little or no mathematical demands of the students.

Each year universities need an adequate supply of new undergraduates with sufficient skill and confidence in mathematics in addition to their main subject requirements. At the moment the school curriculum is not providing them. The number taking AS/A Level Mathematics (around 60 - 70 000 a year) is nowhere near sufficient. The remaining new undergraduates have done no mathematics for the last two years and so arrive with the qualification of GCSE Mathematics This is a particularly English & Welsh problem; almost everywhere else in the world all school students continue with mathematics to the age of 18.

Two factors have made the situation worse in recent years. The first is the decline in the numbers taking A Level Mathematics, by about 40% since the early 1980s. The second is the increasingly mathematical nature of most academic work.

The new AS/A Level curriculum

This September students started on new AS/A Level courses and they are embedded in a new curriculum. Sixth form students are expected to take 4 or 5 subjects to AS Level in the first year and then concentrate on 3 in the second. At first sight it would seem that this opens the door to much wider participation in mathematics with many students taking it as one of the extra AS subjects.

We will have to wait some years before we know whether that actually happens, but there are reasons to question whether it will.

Mathematics was the first subject to adopt modular A Levels. The MEI syllabus has now been running for some 10 years and the changes for this September are almost non-existent. Although there is to be a new AS standard, no reduction in content has been allowed (by QCA) for the early modules (if anything the reverse), and no change in the standard of the module examination papers. While this policy would be perfectly reasonable if mathematics were to be taken in isolation, it seems set to cause two major problems in the context of the whole new curriculum.

The first of these is that in most other subjects the AS Level has been made distinctly easier. There is plenty of evidence that mathematics is already harder than other subjects at A Level, and this difference is now to be increased in the new AS Levels.

The other problem arises from the expectation that students will take AS Level in one year. With up to ten years of experience of running modular mathematics courses, schools have had time to find the best pattern of entry. With average students, it has turned out to be 2 modules in the first year and 4 in the second. Mathematics departments are now being told by their senior management to do 3 modules instead of their normal 2 in the first year. Furthermore, because most schools are planning a curriculum with 4 or 5 subjects rather than 3 in the first year, the time for each subject is often being reduced.

It is thus predictable that there will be a reduction in the quality of learning in mathematics, and in the grades students obtain, as they are rushed through the early A Level work. Students will see mathematics as harder than other subjects, and they will know that they can expect poor results if they are brave enough to take it.

If so, we can expect no great increase in numbers taking the subject post-16, and quite possibly the reverse. To base policy on the idea that the new curriculum will improve the situation in mathematics would involve an act of faith that it would be hard to justify.

Groups of students

Since it is at the least open to question whether the new curriculum will, in itself, bring any significant increase in mathematics uptake, it is going to be important to encourage students to take at least some mathematics post GCSE. Clearly giving UCAS points for individual modules would do just that. There are in fact two groups of students who are likely to find themselves with one or two mathematics modules.

The first group are those who take GCSE a year early at the end of Year 10 and then do one or two AS/A Level modules along with the rest of their GCSE subjects in Year 11. Some of these will of course go on to do A Level Mathematics but others will switch to other A Levels being left with modules which, it is proposed, will carry no credit. The practice of entering GCSE early, or of taking another qualification alongside it, is of course long established.

The second group are those who set out to do AS in mathematics in the first year of sixth form as a fourth or fifth subject and find that their overall workload is such that their main subjects are suffering. For example a student intending to read medicine would almost certainly give a higher priority to obtaining grade A in each of physics, chemistry and biology than to completing AS in another subject. Since the final A Level standards in all subjects will be the same as at present, any extra AS Levels that a student takes involve work that is entirely additional to the existing sixth form requirements. Many students who would have coped with the present curriculum will find themselves over-stretched by the new one.

Information for admissions tutors

The modules involved will inevitably be the early ones in any of the strands of mathematics and as such cover the basic ideas upon which the later development of the subject depends. It is perhaps worth noting some of the many topics covered in the early modules of, for example, the MEI syllabus .

> *Pure Mathematics 1* includes the early work on calculus (both differentiation and integration).

Statistics 1 includes all the concepts and terms involved in formal hypothesis testing.

Mechanics 1 includes a thorough grounding in Newton's Laws of Motion.

Such topics take students a long way beyond GCSE. Those who have a good grasp of them will be much better placed to understand the mathematics they are likely to meet and need while at university. This in turn will affect their degree prospects.

It is thus the case that information about such modules is relevant to admissions tutors when they are trying to assess whether particular applicants are likely to make successful undergraduates. It follows that they should be given credit in the new tariff.

The effect on student uptake

Underlying this paper is a recognition that as a nation we need more students to study more mathematics, and that this is of crucial importance for Higher Education.

Schools are not able to compel students to take mathematics in the sixth form. Rather it has to be sold to students, competing in the market place against other subjects. Failing to give credit for individual modules on the UCAS tariff will make that process all the more difficult. Students will be asked to take on an AS course which is harder than others knowing that they will receive no UCAS credit if they do not complete it.

Many of the target students will have done Intermediate Tier GCSE. These students face a problem that is particular to mathematics. Although AS and A Levels are supposed to be available for such students, the syllabuses in mathematics are in effect based on the assumption that they have done Higher Tier GCSE. The Subject Criteria (formerly Subject Core) include a list of Assumed Knowledge, much of which is in the Higher Tier syllabus. As a result those coming in to AS or A Level from Intermediate Tier have to do an amount of work that is comparable to a whole module before they even start on the syllabus proper. For them AS Level is equivalent to 4 modules rather than 3. It is thus unlikely, under the proposed arrangements, that those embark on a one year AS course will end up with any UCAS credit for it, an unattractive proposition indeed.

The situation will however be completely different if single modules attract UCAS credit. Most students appreciate the value and importance of mathematics, and will be persuadable if there is a reasonable prospect of some reward for the time and effort involved.

My prediction is that if single modules are given UCAS credit we will see a substantial increase both in those starting mathematics courses in the first year of sixth form, and also in the numbers taking a single module in Year 11. It will not completely solve the problems faced by Higher Education, but it will do much to alleviate them.

It is sometimes suggested that giving credit for less than the AS qualification is an encouragement for students to give up part way through a course. Those who put forward this argument might pause to consider whether it is preferable for students to learn the content of 2 modules thoroughly, in the given time, or to obtain an insecure and skimpy understanding of 3. They should also realise that a policy of 3 or 0 will result in large numbers of students doing no mathematics at all.

There is another point that needs to be made. Many students arrive at university terrified of mathematics. Their fear of the subject is often more damaging than their lack of knowledge because it renders them almost unteachable. School courses which take students through the work too fast in order to complete an AS will only make this problem worse. It is much better for students to be taught rather less at a pace which allows them to build up their confidence. At least then universities will have something they can build on.

Why Mathematics?

The argument in this paper has been given entirely in terms of mathematics. Should it be extended to other subjects ?

There are a number of respects in which mathematics differs from other subjects, and so it is entirely justifiable to make it into a special case.

- Very large numbers of students need to use mathematics as a service subject. The only equivalent subject is English.

- The number of students taking mathematics beyond GCSE is insufficient to support these needs. Consequently mathematics (or the lack of it) is actually holding many university courses back. Not many other subjects are in this unenviable position.

- The ways in which schools organise mathematics ensure that there are opportunities for students to obtain single modules. Fewer opportunities arise in other subjects. In practice many more students obtain single modules in mathematics than in other subjects. There is thus a difference in scale.

- The new curriculum will place mathematics in the position of being harder and less accessible than other subjects.

There is a case for single modules in other subjects also being given credit but the argument would probably follow rather different lines. It might, for example, be presented in terms of being fair to the students, recognising all their achievements in order to build up accurate profiles of them.

These considerations are important but they are outside the scope of this discussion paper, which concentrates on the issues which are specific to mathematics.

Conclusion

The new tariff opens a rare window of opportunity. A relatively minor decision now will bring very considerable benefits to our students in the future. They will be enabled to make more of their degree courses and so will be better prepared to fulfil their potential during their working lives.

"Higher Still" and the teaching of statistics in Scottish schools

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The "Higher Still" review of post-compulsory school education in Scotland, which was concluded in 1997, has brought about important changes to the structure, content and assessment of mathematics courses. This article focuses on one of the principal outcomes, which is the introduction of more statistics into school mathematics in Scotland. The article begins with an overview of the new structure of Scottish National Qualifications, then discusses the statistics issue in more detail.

General Structures

In Secondary 3 and 4 (average age, 13½ years at the start), almost all pupils in almost all Scottish schools undertake two-year programmes leading to Standard Grade qualifications. Typically, pupils follow Standard Grade courses in 8 subjects. Each subject may be passed at either Foundation or General or Credit Level, though typically pupils will follow combined Foundation/General or General/Credit courses of study. Those pupils who achieve a Credit Level pass in a subject have the opportunity to proceed to a Higher course in that subject, lasting one further year. Up until 1999, pupils with Foundation and General awards might have given up the subject altogether or they might have chosen from a selection of stand-alone modules. Pupils with good Higher passes at the end of Secondary 5 might have opted to leave school for university. Those who stayed on at school for a further year might have had the opportunity to follow a Certificate of Sixth Year Studies course.

The "Higher Still" review, which has been implemented progressively since August 1999, changed things for Secondary 5 and 6 pupils in a number of very important ways. First, the stand-alone modules at this level have largely been replaced by new "National Qualifications" (NQ's). These are Access, Intermediate 1 and Intermediate 2 awards, which are broadly equivalent to Standard Grade awards at Foundation, General and Credit level, respectively, though the new NQ's should be completed in one year. This means, for example, that a pupil who does not achieve a Credit Level pass in Mathematics in Secondary 4 might be able to reach an equivalent standard by achieving a General Level pass in Secondary 4 followed by an Intermediate 2 award in Secondary 5.

Here are some possible routes through a subject. (The third of these routes, though possible on paper, seems highly improbable.)

Secondary 3 & 4		**Secondary 5**		**Secondary 6**
Standard Grade - Credit level	→	Higher	→	Advanced Higher
Standard Grade - General level	→	Intermediate 2	→	Higher
Standard Grade - Foundation level	→	Intermediate 1	→	Intermediate 2

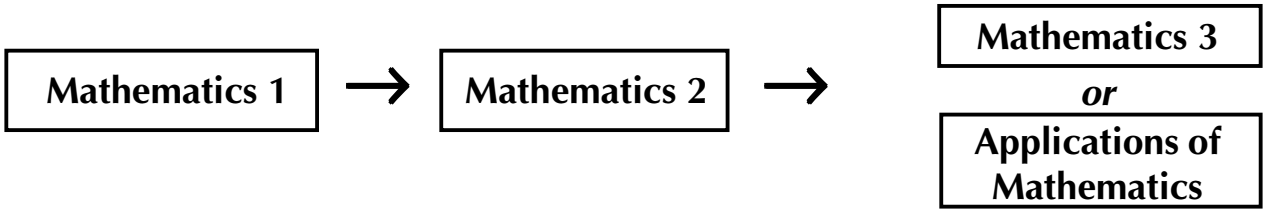
Secondly, the Certificate of Sixth Year Studies has been withdrawn (as of June 2001) and replaced with another new NQ, the Advanced Higher.

Thirdly, all NQ courses (including the Higher) now consist of three units, each rated at 40 hours contact time, with a further 40 hours contact time allowed for consolidation and integration of material. Each unit is internally assessed and the course as a whole is also assessed through an external examination. A pupil must pass all these assessments in order to achieve a course award, although passes in individual units are also certificated.

Statistics in Standard Grade and Intermediate Mathematics

The "Higher Still" review was intended only to affect the courses offered in Secondary 5 and 6. In Mathematics, however, recommendations to change the Standard Grade syllabuses were made and adopted for pupils sitting examinations from May 2001 onwards. From the point of view of possible progression, it is beneficial if the content of the Standard Grade courses is broadly similar to that of the corresponding Access and Intermediate courses. For example, a Higher class might consist of Secondary 5 pupils with Credit passes alongside Secondary 6 pupils with General then Intermediate 2 passes; alignment of the Credit and Intermediate 2 syllabuses ensures that both groups have been exposed to similar mathematical concepts. Prior to the "Higher Still" review, there was no statistical content at all in the Standard Grade Mathematics course, whereas statistics was an important "core" element of the proposed Intermediate courses. Statistical material has now been introduced into Standard Grade to replace the "investigations" that pupils used to carry out. The statistics content of some of these courses is indicated in Table 1.

The topics can be grouped under the general headings of (a) charts and tables, (b) descriptive statistics, (c) scatter diagrams, (d) simple probability.



There are four Mathematics units at both Intermediate 1 and Intermediate 2 – see the above diagram. For a course award, pupils must take the first two units then either Mathematics 3 unit, which is required for progression to higher levels, or Applications of Mathematics, which does not lead to further progression. The statistical content listed in Table 1 comes into the second compulsory unit, with slight extensions in the Applications of Mathematics unit.

Standard Grade (Foundation)

- Complete an ungrouped frequency table and find the mode from it
- Find the mode of a data set
- Complete and extract information from a scattergraph

Standard Grade(General)/Intermediate 1

- Construct and interpret a stem and leaf chart
- Construct a frequency table without class intervals
- Calculate the mean, median and range from a data set and an ungrouped frequency table
- Interpret calculated statistics
- Construct and interpret a scatter graph
- Draw a best-fitting line on a scatter graph (by eye) and use it for prediction
- State the probability of a simple outcome

Standard Grade(Credit)/Intermediate 2

- Construct a pie chart, box plot and dot plot
- Construct a cumulative frequency column for an ungrouped frequency table
- Calculate the quartiles and semi-interquartile range from a data set and an ungrouped frequency table
- Calculate the standard deviation of a data set
- Draw a best fitting line on a scatter graph (by eye), determine its equation and use it for prediction
- Find probabilities in an equally-likely outcomes model

Table 1: Statistics content of Intermediate and Standard Grade courses

Statistics in Higher Mathematics

One of the keynotes of the "Higher Still" review was "minimal change" to the content of Higher courses. In Mathematics, however, there was a strongly-held opinion among some teachers that statistics should be introduced into Higher Mathematics where previously there had been no statistical material. In order to reconcile these two conflicting positions, four units were introduced, three on pure Mathematics and one on Statistics. The three Mathematics units, between them, cover the same material as the previous Higher Mathematics course. Now, however, only the Mathematics 1 and Mathematics 2 units are compulsory and pupils aiming for a course award may take *either* the Mathematics 3 *or* Statistics units.

The content of this optional Statistics unit is shown in Table 2. It is worth pointing out that the emphasis in exploratory data analysis is now on interpreting graphs, tables and descriptive statistics, not on drawing or calculating them. The inferential nature of statistics (drawing conclusions about a population on the basis of sample data) is emphasised here. It is perhaps unusual to find such a strong emphasis on continuous distributions at this level, but this material articulates well with material on differentiation and integration that is introduced in the compulsory Mathematics units.

Unfortunately, pupils who choose the Higher Statistics unit miss out the topics covered in Mathematics 3, which include the logarithm and exponential functions. These are important concepts for budding statisticians (think of the standard continuous distributions) and it is regrettable that pupils should be faced with such an important choice so early in their mathematical education. Those pupils who continue to Advanced Higher Mathematics can fill this gap in their knowledge, but this is only a small proportion of the cohort. It is also difficult for universities to be sure just what material a pupil presenting with a Higher Mathematics qualification will have covered. Some Scottish universities have responded by requiring a course award that includes Mathematics 3 for entry to subjects that are highly mathematical. Perhaps the alternative course, which includes the Statistics (H) unit, will appeal most to pupils who know that their future career lies in a subject where statistics is heavily used, such as Psychology or Biology or Business Studies.

Interpret the results of an EDA
Discrete probability
- using the laws of probability
- probability mass function
- mean and variance
- simple simulation

Continuous random variables
- probability density and cumulative distribution functions
- calculate probabilities from a p.d.f. or c.d.f.
- calculate the mean and variance from a p.d.f.

Relationship between two variables
- determine equation of linear regression
- use linear regression for prediction
- calculate correlation coefficient

Table 2: Content of the Statistics (H) unit

Unit 1

Probability

- conditional probability
- Bayes' Theorem

Random variables

- expected value and variance of sums and differences of two
- random variables
- Binomial, Poisson and Normal distributions
- Poisson and Normal approximations to the Binomial

Sampling

- convenience, quota, simple random, stratified and cluster sampling
- the distribution of the sample mean
- the Central Limit Theorem
- confidence interval for a population mean (variance known)
- approximate confidence interval for a population proportion

Hypothesis tests

- Z test for population mean
- Z test for difference in population means (paired data)

Unit 2

Control charts

- x bar charts
- p charts

Hypothesis testing

- Sign Test
- Mann-Whitney U Test
- Chi-squared goodness-of-fit test (discrete distributions only)
- Chi-squared test of association

Student's t distribution

- t interval for a population mean
- t test for a population mean
- t intervals and tests with paired data

Linear models

- residual plots to check model assumptions
- confidence interval for slope parameter
- confidence and prediction intervals in the linear model
- confidence interval for the population correlation

Table 3: Content of Statistics (AH) units

Statistics in Advanced Higher Mathematics

The Certificate of Sixth Year Studies included a little probability and exploratory data analysis in the General paper and a full, optional Statistics paper. This qualification has been replaced by an Advanced Higher Mathematics and an Advanced Higher Applied Mathematics. In total, there are nine Mathematics modules at Advanced Higher level: Mathematics 1, 2 and 3; Statistics 1 and 2; Numerical Analysis 1 and 2; Mechanics 1 and 2.

The content of the Statistics units is shown in Table 3. A pleasing progression in statistical understanding can clearly be seen by comparing Tables 1 to 3. For example, drawing scatter graphs and fitting lines by eye (Intermediate) gives way to least-squares estimation (Higher) and then to the use of residual plots for model validation (Advanced Higher).

In each of the Advanced Higher units, pupils are also required to undertake an extended coursework task, which is assessed separately from the examination. A course award in Advanced Higher Mathematics requires (i) Mathematics 1 and 2 plus (ii) *either* Mathematics 3 *or* Statistics 1 *or* Numerical Analysis 1 *or* Mechanics 1. A course award in Advanced Higher Applied Mathematics requires (i) a pair of Statistics *or* Numerical Analysis *or* Mechanics units plus (ii) *either* the first unit in one of the other applications of mathematics *or* Mathematics 1. Schools were recently informed that pupils may count the same unit towards a course award for both these Advanced Highers, although the pupil will have to sit the external examination for both courses. This means that a pupil can take just four units, for example, Mathematics 1 and 2 and Statistics 1 and 2, yet still achieve course awards in both Advanced Higher Mathematics (using Mathematics 1 and 2, Statistics 1) and Advanced Higher Applied Mathematics (using Mathematics 1, Statistics 1 and 2).

It is possibly also worth noting that pupils can take the Advanced Higher Statistics units whichever units they chose at Higher level. A new textbook at this level, by Riddiough and McColl (2000), integrates recapitulation of the required Higher Statistics material with coverage of the "new" Advanced Higher material.

Issues – a statistician's perspective

1. From the statistician's point of view, the "Higher Still" review of Mathematics is to be welcomed. In particular, the introduction of basic statistical material into the Standard Grade and Intermediate syllabuses is extremely beneficial for the general education of Scottish young people.

2. Many young people might benefit from studying Statistics as part of the Higher Mathematics course. This is particularly true of those who are contemplating careers in areas where statistics is routinely used, such as Psychology, Biology, Business Studies. This benefit will only accrue if the optional Statistics (H) unit is offered in all or most schools. At the moment, this seems unlikely to happen. There are substantial resource implications of offering a choice of courses to pupils, especially as Higher Mathematics classes are small in many schools. All but a handful of schools have opted for the "minimum change" strategy of making only the Mathematics units available to pupils.

3. The Advanced Higher Statistics syllabus is exciting, but entry to Statistics units at this level is again likely to be restricted due to lack of resources at the school level. There is also a widespread misunderstanding that pupils may only proceed to Advanced Higher Statistics modules if they have already taken the Statistics (H) unit, but the relevant Higher material may actually be picked up quickly and naturally in the course of studying for the AH units.

4. The vast majority of Scottish teachers have no formal training in statistics and therefore lack the confidence to teach statistical material at Higher or Advanced Higher level. In order for maximum benefit to be gained from these recent changes, it will be necessary to invest in teacher training. Even then, the cost of providing choices to pupils may be prohibitive.

5. Universities face considerable uncertainty in establishing First Year courses that will articulate with all the possible combinations of units that pupils may experience at school. They will be required to reflect on whether it is appropriate to bar entry to university courses to prospective students who have demonstrated mathematical ability in one of these possible combinations rather than another. Such a policy might appear unjust, especially if school pupils have no say in which mathematics units they take.

References

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AS: Use of Mathematics

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From September 2001, the new GCE AS *Use of mathematics* will be nationally available for teaching with the first full award from summer 2002. This new qualification has been introduced by the Government with the hope that its availability "will encourage more students than at present to continue with appropriate study of mathematics beyond the end of compulsory schooling at age 16".

Use of mathematics is designed for students with GCSE Mathematics at grade C or above who do not intend to take the subject as far as GCE A level Mathematics, yet who would wish to understand and apply mathematics to a range of real situations.

Design Principles

This new qualification is designed to "promote the application of mathematical principles to analyse real situations and solve problems and to be of value to learners following 'academic', 'vocational' and 'mixed' study programmes" using the following principles:

- Mathematical principles should be introduced through *applications*
- Links are made with students' studies, work or interests: what mathematics is used and how is it used?
- Use of ICT is thoroughly integrated in the form of, for example, graphical calculators or spreadsheets
- Mathematical communication skills are developed
- The solution of substantial problems is promoted
- Assessment is by methods that promote the above

The structure of the qualification

Use of mathematics has three components.

1. The unit "Working with algebraic and graphical techniques" which is mandatory.
2. Students then chose from two options "Modelling with Calculus" and "Using and Applying Statistics".
3. The last component is titled, "Applying mathematics" and is known as the terminal unit.

The units in parts (1) and (2) are available as Free-Standing Maths Qualifications (FSMQs) and the terminal unit is unique to this qualification.

Assessment

The assessment of the FSMQ components comprises two elements: a coursework portfolio and a written examination. In the Portfolio students are expected to demonstrate their use of mathematics in substantial pieces of work linked to their other studies/activities. Assessment of the FSMQs will involve structuring & presenting work, using appropriate mathematics accurately and interpreting mathematics. Students are expected to have access to graphic calculators in all the written examinations. To complete the AS Use of Mathematics students also need to take the terminal unit, the assessment of which will be entirely by written examination, including a comprehension paper. Students who decide only to complete the FSMQs will still receive certificates and UCAS points for those FSMQs. Students awarded the AS *Use of mathematics* will also be awarded level 3 Application of Number as the qualification is proxy for the whole of this key skill.

We now briefly describe the content of the four units - the three FSMQs - the students take the first and one of the other two - and the mandatory terminal unit.

FSMQ - Working with Algebraic & Graphical Techniques

Mathematical content: Functions - powers of x, exponential, trigonometric. Solution of quadratic, polynomial, trigonometric equations. Use of logarithms.

Portfolio requirements: Reports of investigations in which functions are used as models of real data - including evidence of graphical interpretation and use of algebra. Use of logarithms or other linearisation techniques to fit functions to non-linear data.

FSMQ - Modelling with Calculus

Portfolio requirements: Reports of investigations demonstrating the use of: differentiation, integration, numerical methods, differential equations.

FSMQ - Using & Applying statistics

Mathematical content: Measures of location and spread, distributions - the 'normal' used to model data, correlation and regression, significance.

Portfolio requirements: Use of statistics to make sense of a situation including: identification of the purpose of a study and appropriate data, selection and use of statistical measures and diagrams, conclusions and critical consideration of limitations, communication of findings in non-technical language. Critical analysis of the statistical work of others.

Terminal Unit

This unit extends use of mathematics to model situations using algebra, functions and graphs. It introduces simulations of random events and recurrence relations to model situations and aims to develop skills to enable critical analysis of the use of mathematics and the communication of mathematics.

The objectives are

1. To develop in students an understanding of how mathematics can be used to model different situations. This includes the modelling cycle.
2. To use simulation techniques to develop mathematical models for random events.
3. To extend, develop and use a range of numerical, algebraic and graphical techniques when modelling situations. For example, appreciation of algebraic, graphical and numerical methods, use of algebra to find solutions and verify that solutions are valid, solution of systems of equations solution of linear inequalities, use of recurrence relations in discrete models.
4. To appreciate that general mathematical principles may be applicable in a range of different contexts: for example to be able to identify the use of particular mathematical ideas across a range of situations or contexts and to understand when mathematical methods will lead to solutions. Students should appreciate general graphical features including intercepts with axes and asymptotes and develop understanding of the use of geometric transformations applied to graphs of basic functions.
5. To "make sense of mathematics". This includes being able to explain steps in mathematical working, the relation of mathematics in new situations to mathematics in more familiar situations and to develop strategies to assist in making sense of mathematics.
6. To work accurately, structuring mathematical arguments carefully and communicating mathematics clearly: This includes identification of possible errors in mathematical working, use of appropriate degree of accuracy, writing clear and unambiguous mathematical statements, use of correct mathematical notation, use of a range of algebraic, graphical and numerical approaches to communicate mathematics effectively

More information about this qualification, including example papers and materials, can be found at
http://www.qca.org.uk/nq/subjects/as_use_mathematics.asp

More information on the FSMQs can be found at exam board websites, www.aqa.org.uk, www.edexcel.org.uk and www.ocr.org.uk